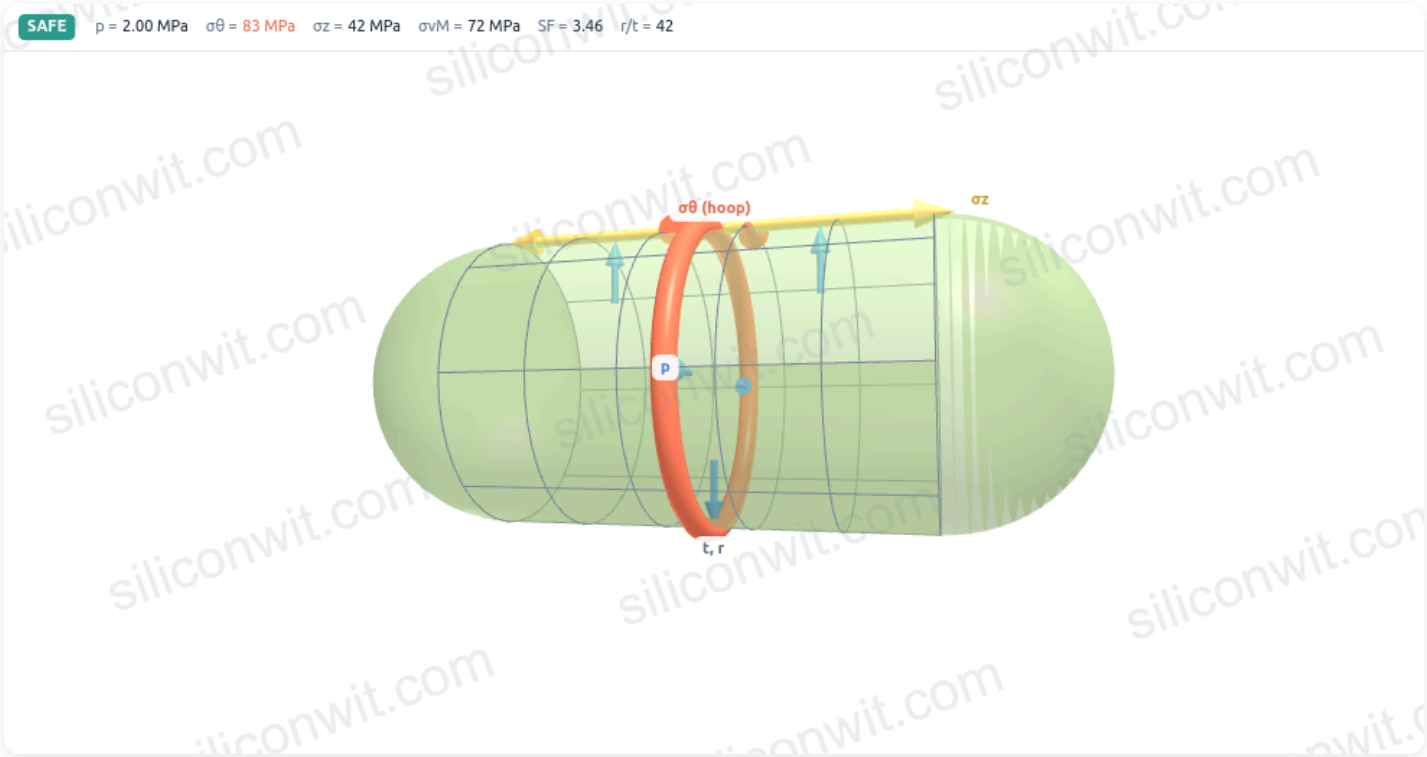
Lesson 1.6: Thin-Walled Pressure Vessels



Modified: Aug. 2, 2026

Published: Sep. 16, 2025



Become Contributor

★ Expert Support

A pneumatic actuator casing that is too thin ruptures under pressure; one that is too thick wastes material, adds weight, and costs more. Unlike a solid bar in tension, a pressurized cylinder develops two distinct stresses simultaneously: hoop stress around its circumference and longitudinal stress along its axis. The hoop stress is always twice the longitudinal, which is why cylindrical vessels split lengthwise rather than across. A

sphere, by contrast, carries the same stress in every direction at the wall, and that symmetry means it needs only half the wall thickness of a cylinder at the same pressure. In this lesson you will calculate both stresses, size wall thickness for a given pressure and material, and see the sphere advantage proved by numbers. #PressureVessels #HoopStress #PneumaticDesign

Learning Objectives

By the end of this lesson, you will be able to:

1. **Derive** the hoop and longitudinal stress formulas for a thin-walled cylinder from a free-body cut
2. **Calculate** both stresses in a cylindrical pressure vessel and identify which one governs the design
3. **Size** the minimum wall thickness for a given internal pressure and allowable stress
4. **Evaluate** the safety factor of an existing wall against the material yield strength
5. **Explain** why a spherical vessel carries wall stress in all directions equally and requires about half the wall material of an equivalent cylinder
6. **Compute** the circumferential and longitudinal strains and the resulting changes in diameter, length, and volume of a pressurized cylinder
7. **Size** the wall thickness of a seamed vessel using a joint efficiency and a factor of safety on the ultimate strength

> **Try it as you read**
>
> This lesson pairs with the Pressure Vessel Simulator: pressurize a thin-walled cylinder and watch the hoop stress come out twice the longitudinal, then switch to a sphere and see the wall stress halve. The Pressure Vessel Experiments reproduce the worked examples step by step with Python.

Real-World System Problem: Pneumatic Actuator Casing



Pneumatic actuators drive grippers, clamps, slides, and valves throughout industrial automation. The cylindrical casing contains pressurized air at every working moment, so a stress error in the wall is not a performance issue but a safety issue.

The Pressure Vessel Problem

> **Engineering Question:** How do we determine the minimum wall thickness for a pneumatic actuator casing operating at 0.6 MPa gauge pressure, and what stresses does that wall actually carry?

Without a stress model, “thick enough” is guesswork. With it, you can choose the wall thickness, predict the failure mode (lengthwise splitting, not end-cap blowout), pick the right safety factor, and prove compliance to a pressure-vessel code.

Why Pressure Vessel Analysis Matters

Predicting failure mode

The 2:1 ratio between hoop and longitudinal stress tells you the vessel splits lengthwise first. Getting the ratio wrong means the wrong weld seam orientation, the wrong inspection plane, and an unconservative design.

Sizing wall thickness

Because hoop stress governs, the minimum wall thickness comes directly from the hoop formula. Setting that thickness correctly prevents both under-design (burst) and over-design (excess mass and cost).

Material and geometry tradeoffs

Cylinder versus sphere at the same pressure and radius: the sphere wall stress is half the cylinder hoop stress, so the sphere needs half the wall. Knowing this guides vessel shape selection from the start.

Code compliance

Pressure vessel standards (ASME, ISO 4393) mandate specific safety factors. The analysis here is the calculation the code is built on, and you cannot fill in a compliance form without it.

Fundamental Theory: Pressure Vessel Mechanics

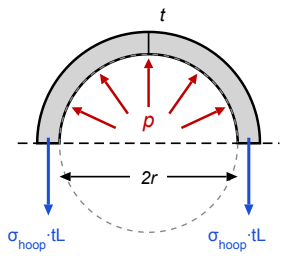
Thin-Wall Assumption

The thin-wall formulas apply when the radius-to-thickness ratio satisfies:

$$\frac{r}{t} \geq 10$$

Below this ratio the stress varies noticeably across the wall thickness and thick-wall (Lame) equations are needed. Above it, the stress is nearly uniform across the wall and the two simple formulas below are accurate to within a few percent.

(a) Hoop stress: cut the cylinder lengthwise



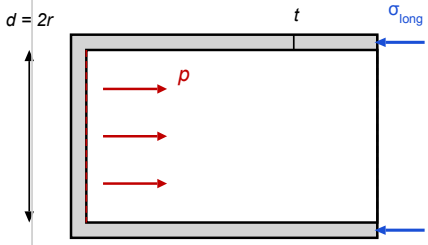
projected area = 2rL
(L = length into the page)

Equilibrium of the half shell:

$p\,(2rL) = 2\,\sigma_{hoop}\,(tL)$

$\sigma_{hoop} = p\,r\,/\,t$

(b) Longitudinal stress: cut across the axis



pressure acts on the end area = πr²

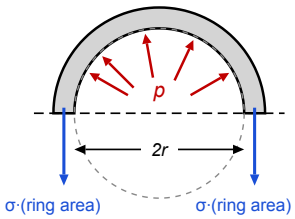
resisted by the wall annulus = 2πrt

Equilibrium of the cut-off portion:

$p\,(\pi r^2) = \sigma_{long}\,(2\pi rt)$

$\sigma_{long} = p\,r\,/\,2t$ **(half the hoop)**

(c) Sphere: any diametral cut is the same



projected area = πr²
resisted by the ring area 2πrt

Equilibrium of the hemisphere:

$p\,(\pi r^2) = \sigma\,(2\pi rt)$

$\sigma = p\,r\,/\,2t$ **(in every direction)**

Each formula below is one of these free bodies. Nothing is memorised: cut the vessel, ask what area the pressure pushes on and what area of wall resists it, and the stress follows from force balance alone.

Hoop (Circumferential) Stress in a Cylinder

Cut the cylinder lengthwise through a diameter. Internal pressure p acts on the projected area $2rL$ and is resisted by two wall cross-sections each of area tL:

$$\sigma_{hoop} = \frac{p\,r}{t}$$

Where p is gauge pressure (Pa), r is the internal radius (m), and t is the wall thickness (m). This is the larger of the two principal stresses and it governs the design.

Longitudinal (Axial) Stress in a Cylinder

Cut the cylinder across its axis. Pressure p acts on the end-cap area πr^2 and is resisted by the annular wall area $2\pi rt$:

$$\sigma_{long} = \frac{p\,r}{2t}$$

This is exactly half the hoop stress. The cylinder is therefore in biaxial stress: $\sigma_{hoop} = 2\,\sigma_{long}$.

Stress in a Spherical Vessel

Any diametral cut through a sphere gives a symmetric result. Pressure p acts on the projected area πr^2 and is resisted by the wall area $2\pi rt$, so the same formula applies in every direction:

$$\sigma_{sphere} = \frac{p\,r}{2t}$$

The sphere wall stress equals the cylinder longitudinal stress and is half the cylinder hoop stress. For the same pressure, radius, and wall thickness, the sphere wall is loaded to half the stress of the cylinder wall.

Volumetric Strain and Change in Dimensions

A pressurized cylinder wall is in biaxial stress, so each strain follows Hooke's law with a Poisson contribution from the perpendicular stress:

$$\varepsilon_{hoop} = \frac{\sigma_{hoop} - \nu\,\sigma_{long}}{E} = \frac{pr}{tE}\left(1 - \frac{\nu}{2}\right), \qquad \varepsilon_{long} = \frac{\sigma_{long} - \nu\,\sigma_{hoop}}{E} = \frac{pr}{2tE}\left(1 - 2\nu\right)$$

The circumferential (hoop) strain is exactly the diametral strain, so the diameter and length change by:

$$\Delta d = \varepsilon_{hoop}\, d, \qquad \Delta L = \varepsilon_{long}\, L$$

A volume grows in two circumferential directions and one axial direction at once, so the volumetric strain is the sum of two hoop strains and one longitudinal strain:

$$\varepsilon_V = 2\,\varepsilon_{hoop} + \varepsilon_{long}, \qquad \Delta V = \varepsilon_V\, V, \quad V = \pi r^2 L$$

For a sphere every direction carries the same stress $\sigma = pr/2t$, so the strain is $\varepsilon = \dfrac{\sigma(1-\nu)}{E}$ in every direction and the volumetric strain is $\varepsilon_V = 3\varepsilon$ with $V = \frac{4}{3}\pi r^3$.

Joint Efficiency and Design Thickness

A real vessel is rolled from plate and closed by a riveted or welded seam that is weaker than the parent plate. A joint efficiency η (a value between 0 and 1) is the fraction of the solid-plate strength the seam retains. Because the longitudinal seam runs along the axis, it carries the hoop stress and governs the wall thickness:

$$t_{min} = \frac{p\,r}{\sigma_{allow}\,\eta}, \qquad \sigma_{allow} = \frac{\sigma_{UTS}}{SF}$$

The circumferential seam carries only the longitudinal stress, so it needs $t = pr/(2\,\sigma_{allow}\,\eta_c)$ and rarely governs. A perfect joint ($\eta = 1$) recovers the solid-wall result $t_{min} = pr/\sigma_{allow}$.

Application 1: Pneumatic Actuator Casing

A compact pneumatic gripper casing is the starting point: a cylinder that must contain working pressure while staying light enough to be end-of-arm tooling.

System Problem Statement

- **Component:** aluminum 6061-T6 pneumatic actuator casing (cylindrical)
- **Working pressure:** $p = 0.6$ MPa gauge
- **Internal diameter:** 100 mm, so radius $r = 50$ mm
- **Material:** aluminum 6061-T6, $\sigma_{yield} = 270$ MPa
- **Required safety factor:** SF = 4.0
- **Task:** find the minimum wall thickness, then verify the actual stresses with a chosen standard thickness of 3.0 mm

Key Question: What wall thickness does the pressure require, and how large is the safety margin with a 3 mm wall?

Step 1: Minimum Wall Thickness from Hoop Stress

> **Click to reveal the wall thickness calculation**

Step 2: Verify Actual Stresses with t = 3.0 mm

> **Click to reveal the stress verification**

Step 3: Interpret the Design

> **Click to reveal the design interpretation**

> **Engineering Insight**
>
> The minimum calculated thickness tells you the physics minimum; the selected thickness is decided by manufacturing constraints, stress concentrations, and code minimums. Both numbers matter, and they usually differ by a large factor in low-pressure pneumatics.

Application 2: Compressed-Air Receiver Tank

A compressed-air receiver stores energy between the compressor and the distribution network. It is a plain cylindrical vessel at a higher pressure than typical pneumatic actuators, with no ports other than an inlet and an outlet, so the nominal wall stress is the governing calculation.

> **System Problem Statement**
>
> - **Component:** steel receiver tank (cylindrical pressure vessel)
> - **Internal pressure:** $p = 1.6$ MPa gauge
> - **Internal diameter:** 400 mm, so radius $r = 200$ mm
> - **Wall thickness:** $t = 10$ mm (as manufactured)
> - **Material:** carbon steel, $\sigma_{yield} = 350$ MPa
> - **Task:** calculate the actual hoop and longitudinal stresses, check the safety factor against yield, and then find the minimum wall thickness for SF = 4.0
>
> **Key Question:** Is the 10 mm wall adequately safe, and how thin could the wall theoretically be at this pressure?

Set this cylinder in the simulator and read the hoop and longitudinal stresses and the safety factor:

Open the Pressure Vessel Simulator

Step 1: Verify the Thin-Wall Assumption

> **Click to reveal the thin-wall check**

Step 2: Hoop and Longitudinal Stress

> **Click to reveal the stress calculations**

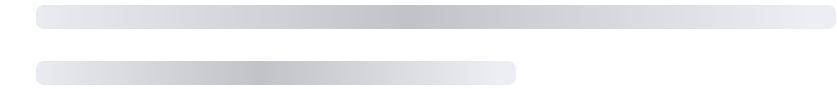
Step 3: Safety Factor and Minimum Wall Thickness

> **Click to reveal the safety factor and wall sizing**

Engineering Insight

A safety factor of 10.9 against static yield on a receiver tank is not over-design. Compressed-air receivers operate continuously for decades, accumulating millions of pressure cycles as the compressor loads and unloads. Fatigue, corrosion, and occasional pressure spikes all erode the apparent margin, and the 10 mm standard wall reflects those realities.

Application 3: Spherical Gas Storage Vessel



Large gas storage spheres appear at refineries, LPG terminals, and compressed-gas facilities. The sphere shape is chosen for a direct mechanical reason: at the same internal pressure and radius, the wall stress in a sphere is exactly half the hoop stress in a cylinder. That means about half the wall thickness for the same duty, which matters when the vessel weighs hundreds of tonnes.

System Problem Statement

- **Component:** spherical pressure vessel (e.g. a hydraulic accumulator shell or gas storage sphere)
- **Internal pressure:** $p = 1.6$ MPa gauge (same as Application 2)
- **Internal radius:** $r = 200$ mm (same as Application 2)
- **Wall thickness:** $t = 10$ mm (same wall for direct comparison)
- **Material:** carbon steel, $\sigma_{yield} = 350$ MPa
- **Task:** compute the sphere wall stress, compare it with the cylinder hoop stress from Application 2, and find the minimum wall thickness the sphere needs at SF = 4.0

Key Question: By how much does changing from a cylinder to a sphere reduce the wall stress and the required wall thickness?

Step 1: Sphere Wall Stress

> **Click to reveal the sphere stress calculation**

Step 2: Safety Factor for the Sphere

> **Click to reveal the sphere safety factor**

Step 3: Minimum Sphere Wall Thickness at SF = 4.0

> **Click to reveal the minimum wall comparison**

Engineering Insight

The sphere’s material efficiency comes entirely from geometry. Every point on a spherical shell has the same curvature in every direction, so internal pressure is resisted equally in all meridional planes. A cylinder resists pressure efficiently around its circumference (hoop) but much less efficiently along its axis (longitudinal), because only the narrow annular wall area carries the end-cap load. That geometric imbalance is the whole story behind the 2:1 stress ratio and the 2:1 wall-thickness ratio.

Application 4: Steam Drum Strains, Volume Change, and a Riveted Seam

The first three applications sized walls and compared shapes. A steam drum adds two questions a plant engineer must answer: how much does the drum actually stretch under pressure (which sets clearances, nozzle alignment, and the fill volume), and does the riveted or welded seam, not the solid plate, decide the wall thickness?

System Problem Statement

- **Component:** steel boiler steam drum (cylindrical)
- **Internal pressure:** $p = 2.5$ MPa gauge
- **Internal diameter:** 800 mm, so radius $r = 400$ mm
- **Length:** $L = 3.0$ m
- **Wall thickness:** $t = 16$ mm (as built)
- **Material:** steel, $E = 200$ GPa, $\nu = 0.3$, $\sigma_{UTS} = 400$ MPa
- **Seam:** longitudinal riveted joint, efficiency $\eta = 75\%$; required safety factor SF = 5 on the ultimate strength
- **Task:** find the two stresses and strains, the change in diameter, length, and volume, and the minimum wall thickness the seam requires

Key Question: How far does the drum grow under pressure, and is the 16 mm wall thick enough once the seam efficiency is accounted for?

Step 1: Stresses and Strains

> **Click to reveal the stress and strain calculation**

Step 2: Change in Diameter, Length, and Volume

> **Click to reveal the dimensional changes**

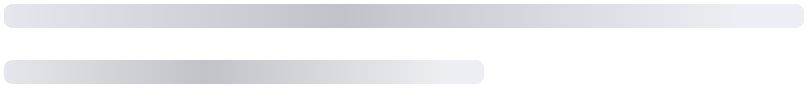
Step 3: Minimum Wall Thickness for the Riveted Seam

> **Click to reveal the seam thickness check**

Engineering Insight

Two lessons sit inside this drum. First, the strains are tiny (a fifth of a millimetre on an 800 mm diameter), yet they are what open flange gaps, misalign nozzles, and set the true fill volume, so they are designed in, not ignored. Second, the joint efficiency can move the governing thickness by a third or more: a vessel that looks safe on the solid-plate formula can still fail at the seam, which is exactly why pressure-vessel codes rate the joint separately.

Design Guidelines for Thin-Walled Pressure Vessels



Always use radius, not diameter

The formulas $\sigma = pr/t$ and $\sigma = pr/(2t)$ take the internal radius r, not the diameter d. Using d directly doubles the calculated stress. Check your substitution before any other step.

Hoop stress governs cylinder design

In a cylinder, size the wall for hoop stress. The longitudinal stress will then automatically have a safety factor twice as large. A vessel designed only for longitudinal stress is under-designed by a factor of two.

Verify the thin-wall assumption

Confirm $r/t \geq 10$ before using these formulas. Below that ratio the stress gradient across the thickness matters and the simple formulas give an optimistic (unconservative) result.

Select standard wall with practical margin

The physics minimum from $t = pr/\sigma_{allow}$ is a lower bound. Add corrosion allowance (1 to 2 mm for steel), check code minimums, and verify that the next standard sheet thickness clears all stress concentrations around ports and welds.

Rate the seam, not just the plate

A riveted or welded joint is weaker than the parent plate. Divide the allowable stress by the joint efficiency η (or multiply the required thickness by $1/\eta$) so the seam, which usually governs, is the part you actually design for.

Design the growth in, do not ignore it

The strains are small but real. Use $\Delta d = \varepsilon_{hoop} d$ and $\Delta L = \varepsilon_{long} L$ to check flange gaps, nozzle alignment, and fill volume through $\Delta V = \varepsilon_V V$ before committing the design.

> **Instrument it**
>
> A pressure vessel is one of the easiest systems to instrument fully. A pressure transducer reads the internal pressure continuously, and a foil strain gauge bonded circumferentially reads hoop strain directly: divide by Young's modulus and you have the actual hoop stress to compare against your hand calculation. The longitudinal gauge should read half the hoop value, confirming the 2:1 prediction. Wire both gauges into an ESP32 or STM32 and stream the live stress state to siliconwit.io to set an alert well below the burst limit.

Summary and Next Steps

Key Concepts Mastered

1. **The thin-wall assumption** applies when $r/t \geq 10$, making stress uniform across the wall and the simple formulas valid.
2. **Cylinder stresses** are $\sigma_{hoop} = pr/t$ (circumferential) and $\sigma_{long} = pr/(2t)$ (axial).

   Hoop stress is twice longitudinal and governs the design; cylinders fail by splitting lengthwise.

3. **Sphere stress** is $\sigma_{sphere} = pr/(2t)$ in every direction, equal to the cylinder longitudinal stress and half the cylinder hoop stress. A sphere therefore needs half the wall thickness of a cylinder at the same pressure, radius, and safety factor.
4. **Wall thickness sizing** follows from $t_{min} = pr/\sigma_{allow}$ for a cylinder or $t_{min} = pr/(2\sigma_{allow})$ for a sphere, where $\sigma_{allow} = \sigma_{yield}/SF$.
5. **Strains and dimensional change** come from biaxial Hooke's law: $\varepsilon_{hoop} = (\sigma_{hoop} - \nu\sigma_{long})/E$ and $\varepsilon_{long} = (\sigma_{long} - \nu\sigma_{hoop})/E$, giving $\Delta d = \varepsilon_{hoop} d$, $\Delta L = \varepsilon_{long} L$, and $\Delta V = (2\varepsilon_{hoop} + \varepsilon_{long})V$.
6. **Joint efficiency** reduces the seam strength: the governing thickness is $t_{min} = pr/(\sigma_{allow}\,\eta)$, so a riveted or welded seam usually decides the wall, not the solid plate.

Results at a Glance

Application	Geometry	σ_{hoop} or σ_{sphere}	σ_{long}	SF (as built)	t_{min} at SF = 4
Pneumatic actuator casing	Cylinder	10.0 MPa	5.0 MPa	27.0	0.44 mm
Compressed-air receiver	Cylinder	32.0 MPa	16.0 MPa	10.9	3.66 mm
Spherical accumulator	Sphere	16.0 MPa	16.0 MPa	21.9	1.83 mm

The sphere at the same p, r, and t as the receiver tank carries half the wall stress and needs half the minimum wall thickness. Application 4 (the steam drum) adds the deformation and seam picture: at $p = 2.5$ MPa the 800 mm drum grows by

$\Delta d = 0.213$ mm and $\Delta V \approx 895$ cm³, and its 75% riveted seam raises the required wall to 16.7 mm, above the 12.5 mm the solid plate alone would need.

A Note on Tools

All three calculations used the same two formulas and arithmetic with millimetres and megapascals. For a new vessel design, a spreadsheet covering p, r, t, σ_{yield}, and the chosen SF will generate both stresses and both minima in one row. Finite-element analysis is not needed for thin-wall vessels under uniform internal pressure; the hand calculation is the design, and FEA only adds value near ports, welds, and other geometric discontinuities.

This lesson closes Chapter 1. The principles here (force divided by a resisting area, a stress compared against an allowable, a geometry that determines which direction carries more load) carry directly into the beam problems in Chapter 2. The next step is Shear Force and Bending Moment in Beams, where internal pressure is replaced by transverse loading and the resisting area becomes a cross-section in bending.

Comments

Loading comments...

© 2021-2026 SiliconWit®. All rights reserved.

Previous
Lesson 1.5: Torsion of Circular Shafts

Next
Lesson 2.1: Shear Force and Bending Moment

Privacy · Cookie settings