Lesson 1.4: Thermal Stresses and Strains



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A piston that fits perfectly at room temperature can seize in the cylinder at operating temperature if the designer neglected differential thermal expansion between materials. Conversely, a rail that is free to expand causes no stress, but bolt it to a rigid frame and the same temperature rise generates forces large enough to buckle the rail or shear the bolts. In this lesson you will analyze heated piston-cylinder systems and constrained structures, learning to calculate thermal expansion, predict stress in constrained components, and solve compound assemblies where two materials with

different expansion coefficients are bonded at their ends. #ThermalStress #ThermalExpansion #MechatronicDesign

Learning Objectives

By the end of this lesson, you will be able to:

1. **Calculate** free thermal expansion and the thermal strain $\epsilon_T = \alpha \Delta T$ for any material and temperature change
2. **Derive** the thermal stress $\sigma = E\alpha\Delta T$ that develops in a fully restrained member, and check it against yield
3. **Analyze** differential thermal expansion in a piston-cylinder interface to predict hot clearances
4. **Solve** compound (bimetallic) assemblies by writing the compatibility equation and the force-balance equation, then computing the stress in each material

> **Try it as you read**
>
> This lesson pairs with the Thermal Stress Simulator: heat a constrained bar and watch a compressive stress build with no external load, then open a gap and see the stress stay at zero until the bar closes it. The Thermal Stress Experiments reproduce the worked examples step by step with Python.

Real-World System Problem: Heated Piston-Cylinder Interface

In 3D printer extruder heads, injection molding systems, and heated actuators, piston-cylinder interfaces operate at elevated temperatures. Understanding thermal stress is critical for preventing seizure, excessive wear, and component failure.

The Thermal Challenge

> **Engineering Question:** How do we predict the clearance change and the thermal stresses when an aluminum cylinder and a steel piston are heated from room temperature to operating temperature?

Why Thermal Analysis Matters

Seized actuators

If differential expansion closes a running clearance to zero, the piston locks in the bore. The failure is sudden and often requires a full replacement of both parts.

Reduced precision

Even sub-millimetre thermal growth shifts the datum of a positioning stage. In a CNC head or a pick-and-place arm, the part lands in the wrong place.

Hidden stresses

Thermal stresses develop with no applied load. A component that looks lightly loaded at room temperature can be near yield at operating temperature purely from constraint.

Predictable performance

A thermal analysis before the first prototype replaces field failures with deliberate clearances, correct material pairings, and a design that works at every temperature in the range.

Fundamental Theory: Thermal Expansion and Stress



When a material is heated it tends to expand. Whether a stress develops depends entirely on whether that expansion is free or prevented.

Free Thermal Strain and Expansion

Thermal Strain and Expansion

$$\epsilon_T = \alpha \Delta T$$

$$\delta_T = \alpha L \Delta T$$

Where:

- ϵ_T = thermal strain (dimensionless)
- δ_T = free thermal expansion (m or mm)
- α = coefficient of linear thermal expansion (1/°C)
- L = original length (m or mm)
- ΔT = temperature change (°C), positive for heating

A free member (no constraints at either end) expands by δ_T and develops zero stress. The expansion is real but harmless unless it closes a gap or fights another part.

Fully Restrained Member: Thermal Stress

When both ends of a member are rigidly fixed and the temperature rises, the member tries to expand but cannot. The walls push back with a compressive reaction force, and the member develops a compressive stress equal to what Hooke's law would give for that strain:

Thermal Stress in a Fully Restrained Member

$$\sigma_T = E \alpha \Delta T$$

Where:

- σ_T = thermal stress (Pa or MPa), compressive for heating
- E = Young's modulus (Pa or GPa)
- α = coefficient of linear thermal expansion (1/°C)
- ΔT = temperature change (°C)

This result is independent of the member's length. A long rod and a short rod of the same material and cross-section, both fully restrained, develop the same stress for the same temperature rise.

Partial Constraint and Gap Cases

If the member is allowed to expand by a gap g before it contacts a rigid wall, only the excess expansion is restrained:

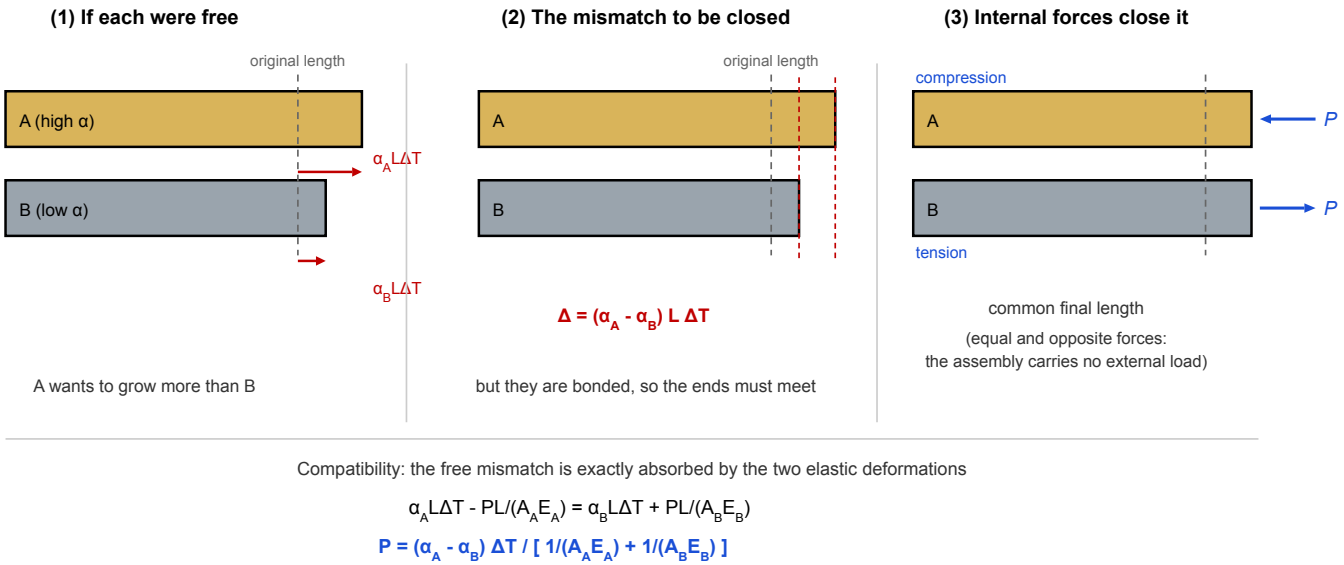
$$\delta_{restrained} = \delta_T - g = \alpha L \Delta T - g$$

$$\sigma_T = E\,\frac{\delta_{restrained}}{L} = E\left(\alpha \Delta T - \frac{g}{L}\right) \qquad \text{valid when } \alpha L \Delta T > g$$

If $\alpha L \Delta T \leq g$ the member never contacts the wall and the thermal stress is zero.

Compound (Bimetallic) Assembly: Compatibility

When two materials with different α values are bonded end-to-end (or inside one another) and heated, they cannot each expand freely because they must stay connected. The system reaches a compromise elongation, and internal forces keep the two parts in equilibrium:



The compatibility equation is easier to write once the problem is seen as three superposed steps: let both expand freely, measure the mismatch, then apply the

internal force pair that closes it. The bar with the larger α ends up in compression and the other in tension, with no external load anywhere on the assembly.

Compatibility Method for Compound Assemblies

Let material A (higher α) and material B (lower α) share the same length L and are bonded at both ends. Both are heated by ΔT.

Compatibility (same total deformation for both): $\delta_A = \delta_B$

Free expansions: $\delta_{A,free} = \alpha_A L \Delta T, \qquad \delta_{B,free} = \alpha_B L \Delta T$

Because $\alpha_A > \alpha_B$, material A tries to expand more. An internal compressive force P develops in A and an equal tensile force P develops in B (Newton's third law).

Compatibility equation: $\alpha_A L \Delta T - \frac{PL}{A_A E_A} = \alpha_B L \Delta T + \frac{PL}{A_B E_B}$

Solving for P: $P = \frac{(\alpha_A - \alpha_B)\Delta T}{\frac{1}{A_A E_A} + \frac{1}{A_B E_B}}$

Then $\sigma_A = P/A_A$ (compression) and $\sigma_B = P/A_B$ (tension).

Application 1: Piston-Cylinder Thermal Analysis

A heated 3D printer extruder uses an aluminum cylinder with a steel piston. This is not a constrained system; the piston moves freely. The analysis predicts whether the running clearance opens or closes with temperature, and what stress would develop if the parts were locked together.

> **System Problem Statement**
>
> - **Aluminum cylinder:** inner diameter $D_{Al} = 20.00$ mm, length 100 mm, material Al 6061 ($\alpha_{Al} = 23.6 \times 10^{-6}$ /°C, $E_{Al} = 70$ GPa, $\sigma_{yield,Al} = 270$ MPa)
> - **Steel piston:** outer diameter $D_{st} = 19.95$ mm, length 80 mm, material Steel 1045 ($\alpha_{st} = 12.0 \times 10^{-6}$ /°C, $E_{st} = 200$ GPa, $\sigma_{yield,st} = 530$ MPa)
> - **Temperature change:** $\Delta T = 180$ °C (from 20 °C to 200 °C)
> - **Cold running clearance:** 0.05 mm
>
> **Key Question:** How does the clearance change at operating temperature, and would either part yield if the expansion were fully constrained?

Step 1: Thermal Expansion of Each Part

> **Click to reveal the thermal expansion calculations**

Step 2: Thermal Stress if Expansion Were Fully Constrained

> **Click to reveal the thermal stress calculations**

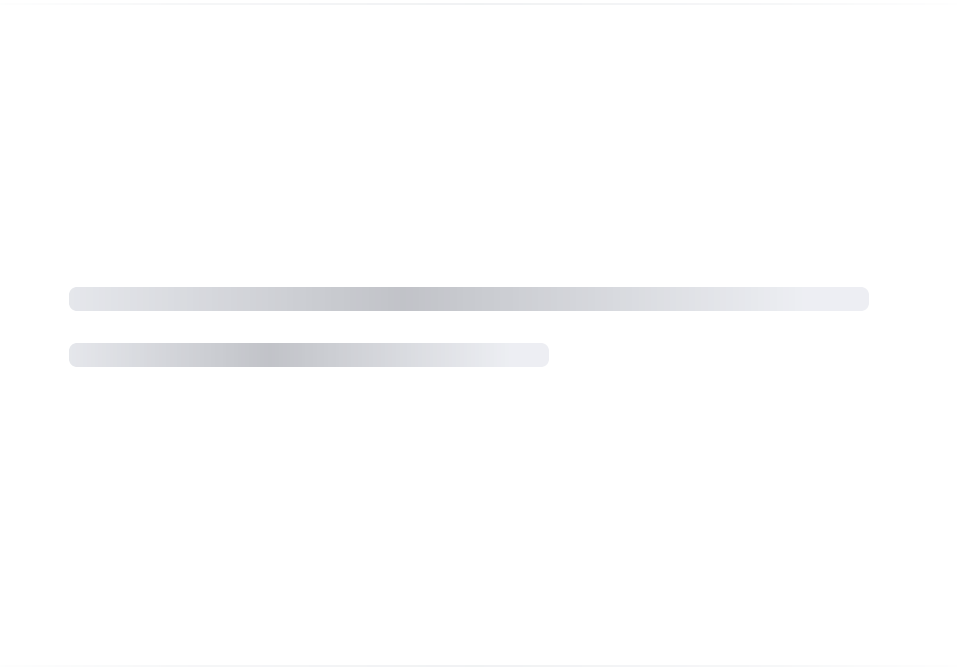
Step 3: Clearance Design for Reliable Operation

> **Click to reveal the design clearance calculation**

Engineering Insight

The direction of differential expansion determines whether a temperature rise opens or closes a running fit. When the bore material has a higher α than the piston material, the clearance grows with temperature. When it is the other way around, the clearance shrinks and seizure becomes the risk. Identifying which case applies is the first thing to check in any heated assembly.

Application 2: Fully Restrained Steel Rod



A steel actuator rod is rigidly clamped at both ends inside an aluminum housing. During a machine warm-up cycle the rod heats up by 60 °C. Because it cannot expand, it develops a compressive thermal stress.

System Problem Statement

- **Component:** steel actuator rod, length $L = 500$ mm, cross-sectional area $A = 200$ mm²
- **Material:** structural steel, $\alpha = 12 \times 10^{-6}$ /°C, $E = 200$ GPa, $\sigma_{yield} = 250$ MPa
- **Temperature rise:** $\Delta T = 60$ °C
- **Boundary condition:** both ends rigidly fixed (fully restrained)

Key Question: What compressive stress develops in the rod, what would the free expansion have been, and is the rod safe against yield?

Clamp a steel bar in the simulator and ramp the temperature to watch the thermal stress climb toward yield:

Open the Thermal Stress Simulator

Step 1: Would-Be Free Expansion

> **Click to reveal the free-expansion calculation**

Step 2: Thermal Stress and Force on the Clamps

> **Click to reveal the thermal stress calculation**

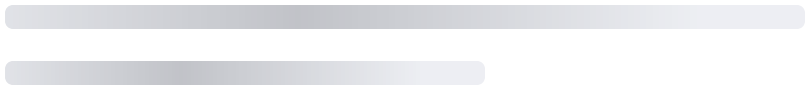
Step 3: Safety Factor Against Yield

> **Click to reveal the safety factor check**

Engineering Insight

A fully restrained member develops $\sigma = E\alpha\Delta T$ regardless of its length. The same temperature rise in a 50 mm rod and a 5000 mm rod of identical material and cross-section gives identical stress. Length determines how large the free expansion would have been, not how much stress the constraint generates. This is why long rails under rigid constraint are more dangerous in the sense that the stored elastic energy is greater, not in the sense of higher stress.

Application 3: Compound Assembly (Aluminum Rod in Steel Housing)



An aluminum rod is pressed into a steel housing and pinned at both ends so the two parts must deform together. When the assembly heats up, aluminum tries to expand more than steel. The two materials fight each other: the aluminum is put in compression and the steel housing is put in tension by the same internal force.

System Problem Statement

- **Aluminum rod:** diameter $d_{Al} = 20$ mm, so $A_{Al} = \pi(20)^2/4 = 314.2$ mm², length $L = 200$ mm, $\alpha_{Al} = 23 \times 10^{-6}$ /°C, $E_{Al} = 70$ GPa
- **Steel housing (tube):** same wall area $A_{st} = 314.2$ mm², length $L = 200$ mm, $\alpha_{st} = 12 \times 10^{-6}$ /°C, $E_{st} = 200$ GPa
- **Temperature rise:** $\Delta T = 80$ °C, both parts heat equally
- **Constraint:** ends pinned; both parts must have the same total elongation

Key Question: What internal force develops, and what stress does each material carry?

Step 1: Free Expansions and the Compatibility Gap

> **Click to reveal the free-expansion and compatibility analysis**

Step 2: Solve for the Internal Force

> **Click to reveal the compatibility equation solution**

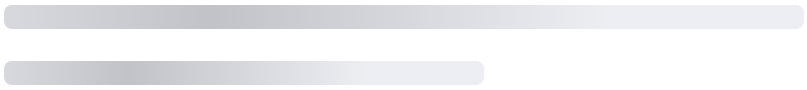
Step 3: Stresses in Each Material

> **Click to reveal the stress results and yield check**

Engineering Insight

The compatibility method generalizes to any number of parallel load paths: write one compatibility equation per redundant constraint and one force-balance equation, then solve simultaneously. The key physical picture is that the high-α material ends up in compression and the low-α material ends up in tension whenever both are heated by the same amount. Reversing which is stiffer changes the magnitude of the stress, not the sign.

Design Guidelines for Thermal Systems



Match expansion coefficients

When two parts must maintain a precise fit across a temperature range, choose materials with similar α values. Steel-to-steel or aluminum-to-aluminum pairings eliminate differential expansion as a problem.

Provide room to grow

Expansion joints, slotted holes, and slip-fit ends let a member expand freely. A free member develops no thermal stress no matter how large the temperature change.

Check constrained stress against yield

Before welding or bolting a member into a rigid frame, compute $\sigma = E\alpha\Delta T$ for the expected temperature range and compare it with the yield strength. If the margin is small, the fix is to allow movement, not to choose a stronger material.

Account for thermal effects in tolerances

Clearances and press-fit interferences are room-temperature numbers. For any heated assembly, recalculate the fit at the operating temperature using the differential expansion before specifying the cold dimensions.

Instrument it

Thermal stress is a temperature problem first. Put a thermocouple (a K-type through a MAX31855) on each part and a dial gauge or LVDT across the clearance, and you can watch the gap close as things heat up and catch the seizure point before it arrives. An ESP32 or STM32 reading temperature and displacement together lets you trend the clearance on siliconwit.io and prove the part expands freely instead of locking up.

Summary and Next Steps

Key Concepts Mastered

1. **Free thermal strain** is $\epsilon_T = \alpha \Delta T$ and free expansion is $\delta_T = \alpha L \Delta T$. A free member develops no stress.
2. **Full constraint** prevents all expansion and develops a compressive thermal stress $\sigma_T = E\alpha\Delta T$, independent of length.
3. **Differential expansion** in a two-material running fit (piston-cylinder) changes the clearance; the direction depends on which material has the higher α.
4. **Compound assemblies** require the compatibility method: write the condition that both parts must reach the same elongation, solve for the internal force, then divide by each area for the stress.

Results at a Glance

Application	Description	Key result	Safety factor
Piston-cylinder (Al/steel)	differential expansion, $\Delta T = 180$ °C	clearance grows from 0.05 mm to 0.092 mm	N/A (free expansion)
Fully restrained steel rod	$\Delta T = 60$ °C, both ends fixed	$\sigma_T = 144$ MPa (compressive), $F = 28.8$ kN	1.74 vs yield
Compound Al rod in steel housing	$\Delta T = 80$ °C, bonded ends	$P = 14.3$ kN; $\sigma_{Al} = 45.6$ MPa (C), $\sigma_{st} = 45.6$ MPa (T)	5.9 (Al), 5.5 (st)

A Note on Tools

All three analyses here used only $\delta_T = \alpha L \Delta T$, $\sigma = E\alpha\Delta T$, and the compatibility equation. No finite-element software is needed to screen a design or size an expansion joint; the hand calculation is the design decision, and simulation only refines geometry once the concept is proven safe.

Next, Fundamentals of Shaft Torsion takes the same material-property framework into rotating shafts, where the load twists the cross-section rather than stretching or compressing it, and the governing stress is shear rather than normal.

Comments

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