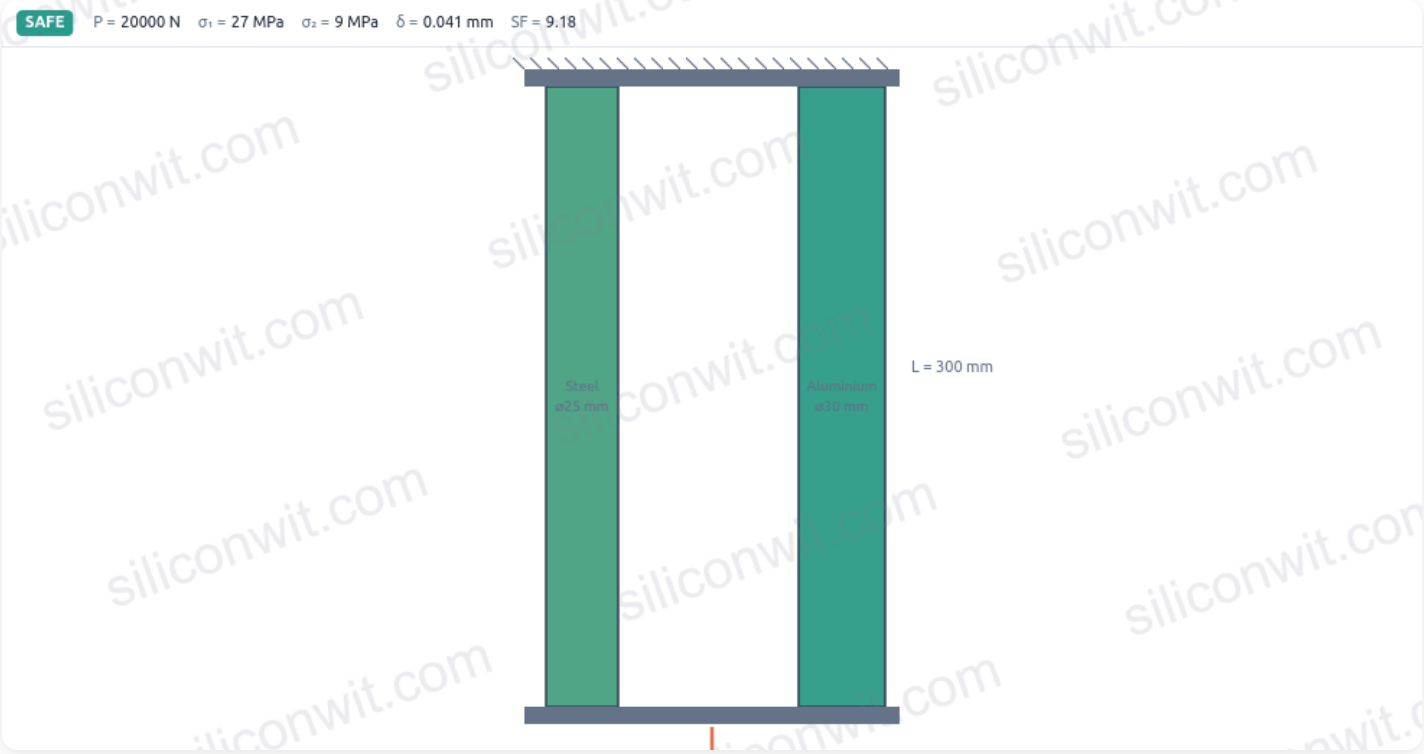
Lesson 1.2: Strain, Material Properties, and Shear in Actuator Systems



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A CNC actuator shaft that stretches even a few micrometres under load throws off positioning accuracy, and choosing a material by cost alone risks either over-designing (heavy, expensive) or under-designing (flexible, imprecise). Selecting the right material

requires comparing elastic modulus, yield strength, and elongation at failure across candidates, then computing the actual deformation under your expected loads. In this lesson you will perform that analysis on a CNC actuator shaft, extend it to members whose cross-section varies along the length (tapered pull-rods, tapered tie plates, stepped bars, bars under their own weight), read a steel tensile test curve to extract yield strength, toughness, and ductility, and compute shear deflection in a bonded elastomer mount. #AxialStress #MaterialSelection #TaperedBars #CNCDesign

Learning Objectives

By the end of this lesson, you will be able to:

1. **Analyze** axial stress and strain in a precision actuator shaft and select between steel and aluminum based on strength and stiffness criteria
2. **Derive and apply** the extension of bars whose section varies along the length (circular tapers, flat tapers, stepped bars) and of bars loaded by their own weight, starting from $\delta = \int N\,dx/(AE)$
3. **Interpret** a tensile test stress-strain curve to identify proportional limit, 0.2% offset yield strength, UTS, fracture strain, modulus of resilience, and modulus of toughness
4. **Apply** the shear modulus relation $G = E/(2(1+\nu))$ to calculate shear stress, shear strain, and lateral deflection in an elastomer component

> **Try it as you read**
>
> This lesson pairs with the Axial Loading Simulator: compare two materials side by side under the same axial load and watch how the stiffness, not the strength, sets the deflection. The Axial Loading Experiments reproduce the worked examples below step by step with Python.

Real-World System Problem: CNC Z-Axis Actuator Shaft

In a CNC machine the Z-axis actuator controls vertical tool movement. The shaft must transmit precise forces while maintaining dimensional accuracy under varying loads. A shaft that deflects too far under the cutting force moves the tool out of the programmed path, producing scrap.

The Shaft Selection Problem

> **Engineering Question:** How do you select the right material and shaft diameter to ensure the CNC system maintains a deformation limit of 0.05 mm under a maximum axial load of 5,000 N?

Answering that requires two independent checks: a strength check (the stress must stay below the allowable strength) and a stiffness check (the elongation must stay within the positional tolerance). Either check can set the minimum diameter, and in precision systems the stiffness check almost always wins.

Why Material Selection Matters

Stiffness governs precision

In positioning systems, the deformation limit is often tighter than the strength limit. A shaft sized purely for strength may deflect several times more than the tolerance allows.

Modulus sets deflection

Steel ($E = 200$ GPa) is about 2.9 times stiffer than aluminum ($E = 70$ GPa). For the same diameter and load, the steel shaft deflects less than a third as much.

Section size follows from E

Because $\delta = FL/(AE)$, a stiffer material needs a smaller area for the same deflection, which keeps the shaft compact.

Mass can be nearly equal

Steel is about 2.9 times denser than aluminum, but steel's higher E lets it use a smaller cross-section, so the final shaft masses can end up similar.

Fundamental Theory: Strain, Hooke's Law, and the Stress-Strain Curve

The previous lesson defined stress. Here we build the quantities needed to predict deformation and to read a material test curve.

Axial Strain and Deformation

Axial Strain and Deflection

$$\epsilon = \frac{\delta}{L_0}, \qquad \delta = \frac{FL_0}{AE}$$

Where:

- ϵ = axial strain (dimensionless)
- δ = change in length (mm or m)
- L_0 = original length
- F = axial force (N)
- A = cross-sectional area (mm² or m²)
- E = Young's modulus (GPa)

The second form is the most-used relation in this lesson: it links a known load and geometry to the actual elongation.

Bars of Varying Cross-Section

$\delta = FL/(AE)$ is only valid when F, A, and E are the same at every section. Real members often are not uniform: a pull-rod is machined with a taper so that material is only where the stress needs it, a tie plate narrows towards the pin, a shaft is stepped for bearing seats, and a long hanging bar carries its own weight. For all of these the load or the area (or both) is a function of position, so the extension has to be built up from an element rather than read off a single formula.

The General Rule for Any Non-Uniform Bar

Take a slice of thickness dx at distance x from the fixed end. The slice is short enough that the internal force $N(x)$ and area $A(x)$ are constant across it, so the uniform-bar result applies to the slice alone:

$$d\delta = \frac{N(x)\,dx}{A(x)\,E}$$

The total extension is the sum of all the slice extensions:

$$\boxed{\delta = \int_0^L \frac{N(x)\,dx}{A(x)\,E}}$$

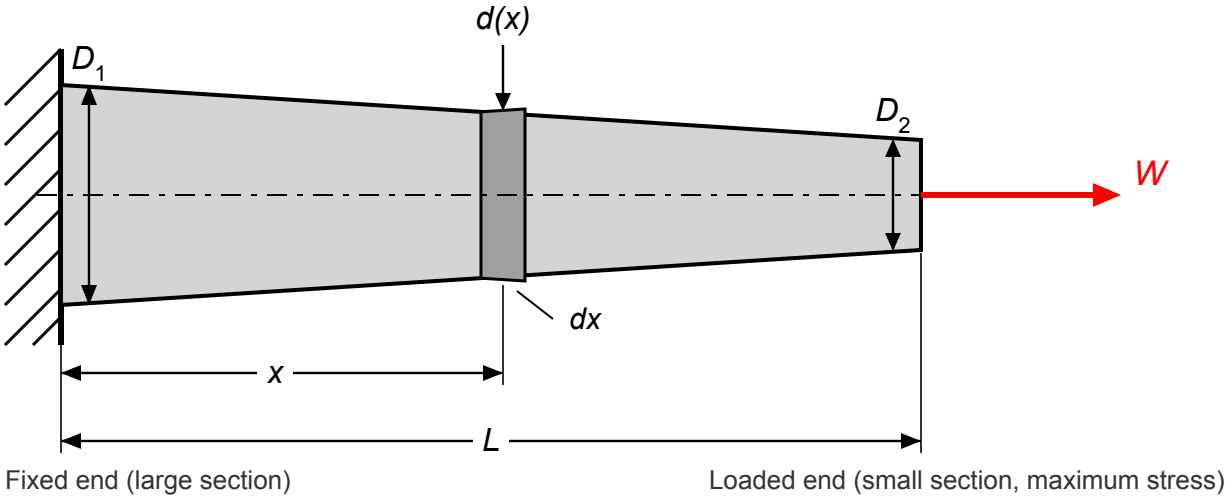
Every result in this section, and every taper, step, or self-weight problem you will meet, comes from this one integral. The uniform bar is just the special case where N and A come out of the integral: $\delta = NL/(AE)$.

The four-step recipe

Work every varying-section problem the same way, and you never have to memorise the individual results:

1. **Write $A(x)$**: the geometry, as a function of distance from one end (linear taper of a diameter, linear taper of a width, etc.)
2. **Write $N(x)$**: the internal force. For an end-loaded bar it is simply the applied load W everywhere; for self-weight it is the weight of the material below the cut.
3. **Integrate $N(x)\,dx/(A(x)E)$** from 0 to L.
4. **Locate the maximum stress separately**: it is at the *smallest area*, and it needs no integration at all: $\sigma_{max} = N/A_{min}$.

Case A: Circular Bar Tapering Linearly (the standard case)



A bar of circular section tapers uniformly from diameter D_1 at the fixed end to D_2 at the loaded end over a length L, carrying an axial load W.

1. **Geometry.** The diameter falls off linearly with x:

$$d(x) = D_1 - \frac{(D_1 - D_2)}{L}x = D_1 - kx, \qquad k = \frac{D_1 - D_2}{L}$$

so the area at any section is $A(x) = \dfrac{\pi}{4}(D_1 - kx)^2$.

2. **Internal force.** The bar is loaded only at its end, so $N(x) = W$ at every section.

3. **Substitute into the general rule:**

$$\delta = \int_0^L \frac{W\,dx}{\frac{\pi}{4}(D_1 - kx)^2 E} = \frac{4W}{\pi E}\int_0^L \frac{dx}{(D_1 - kx)^2}$$

4. **Do the integral.** Since $\dfrac{d}{dx}\left[\dfrac{1}{k(D_1 - kx)}\right] = \dfrac{1}{(D_1 - kx)^2}$:

$$\delta = \frac{4W}{\pi E}\left[\frac{1}{k(D_1 - kx)}\right]_0^L = \frac{4W}{\pi E k}\left(\frac{1}{D_2} - \frac{1}{D_1}\right)$$

5. **Back-substitute $k = (D_1 - D_2)/L$** and combine the fractions,

$$\frac{1}{D_2} - \frac{1}{D_1} = \frac{D_1 - D_2}{D_1 D_2}:$$

$$\delta = \frac{4WL}{\pi E(D_1 - D_2)} \cdot \frac{D_1 - D_2}{D_1 D_2}$$

The taper difference cancels, leaving the standard result:

$$\boxed{\delta = \frac{4WL}{\pi E D_1 D_2}}$$

What the Tapered-Bar Result Tells You

- **Extension:** $\delta = \dfrac{4WL}{\pi E D_1 D_2}$, symmetric in D_1 and D_2, so which end is fixed makes no difference to the total extension.
- **Maximum stress** is at the *small* end, where the area is least: $\sigma_{max} = \dfrac{4W}{\pi D_2^2}$
- **Stress at any section** uses the local diameter: $\sigma(x) = \dfrac{4W}{\pi d(x)^2}$. At mid-length the diameter is the *arithmetic* mean, $d_{mid} = \dfrac{D_1 + D_2}{2}$, giving

$$\sigma_{mid} = \frac{16W}{\pi(D_1 + D_2)^2}.$$

- **Equivalent uniform bar:** setting $\delta = WL/(A_{eq}E)$ gives $d_{eq} = \sqrt{D_1 D_2}$, the *geometric* mean of the end diameters. A uniform bar of that diameter stretches exactly as much as the taper.
- **Cost of tapering:** compared with a uniform bar of the large diameter, the taper stretches D_1/D_2 times as much. Halving the end diameter doubles the extension.

> ⚠ **The most common mistake**
>
> Do **not** compute the extension from the average area or the mid-length area. Extension depends on $\int dx/A(x)$, not on the average of A, and $1/A$ is not linear in x. Using the mid-length section underestimates the extension of a 2:1 taper by 11%, and the error grows as the taper gets steeper. Use the geometric mean $\sqrt{D_1 D_2}$ for a diameter, never the arithmetic mean.
>
> The arithmetic mean $d_{mid} = (D_1 + D_2)/2$ *is* correct for one thing only: the local diameter (and therefore the local stress) at mid-length.

Case B: Flat Bar of Constant Thickness and Tapering Width

A plate of constant thickness t tapers linearly in width from b_1 to b_2 over length L under axial load W. Now $A(x) = t\left(b_1 - \frac{(b_1 - b_2)}{L}x\right)$, and the area appears to the *first* power, so the integral is a logarithm rather than a reciprocal:

$$\delta = \frac{W}{Et}\int_0^L \frac{dx}{b_1 - \frac{(b_1-b_2)}{L}x} = \frac{WL}{Et(b_1 - b_2)}\ln\left(\frac{b_1}{b_2}\right)$$

The equivalent uniform width is the *logarithmic* mean, $b_{eq} = (b_1 - b_2)/\ln(b_1/b_2)$. Maximum stress is again at the narrow end, $\sigma_{max} = W/(t\,b_2)$.

> **Why the two tapers give different-looking answers**
>
> Both come from the same integral. A circular taper varies the diameter linearly, so the *area* varies as the square of a linear function and $\int dx/A$ produces a reciprocal. A flat taper varies the area linearly, so $\int dx/A$ produces a logarithm. Identify how $A(x)$ depends on x before integrating, and the correct form follows automatically.

Case C: Stepped Bar (a Discrete Version of the Same Idea)

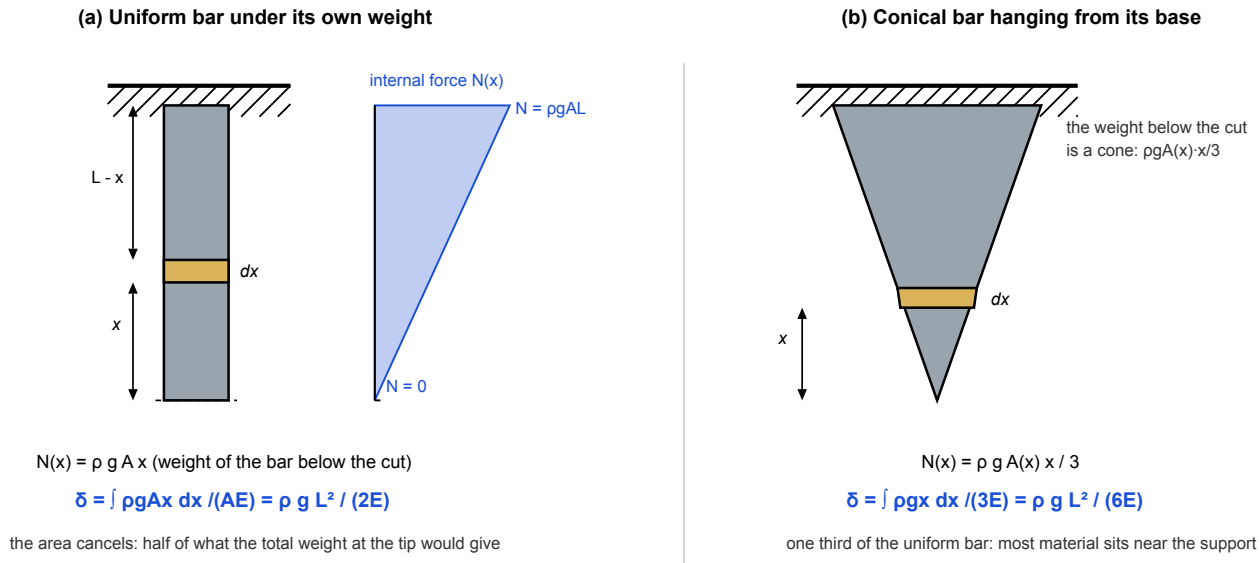
If the section changes in steps rather than continuously, the integral becomes a sum, because each segment is internally uniform:

$$\delta = \sum_i \frac{N_i L_i}{A_i E_i}$$

Each segment is analysed on its own and the extensions are added. The stress is checked segment by segment, and the critical segment is the one with the largest N_i/A_i, which is not necessarily the longest or the most heavily loaded one. Compound bars apply this same summation to segments of different materials.

Case D: When the Internal Force Varies (Self-Weight)

Self-weight problems use the identical integral, except that $N(x)$ varies and A may not:



- **Uniform bar hanging under its own weight** (ρ = density, g = 9.81 m/s²). At a height x above the free end the bar carries the weight below it, $N(x) = \rho g A x$, so

  $$\delta = \int_0^L \frac{\rho g A x \, dx}{AE} = \frac{\rho g L^2}{2E} = \frac{W_{total} L}{2AE}$$

  which is exactly half the extension the total weight would produce if it were hung at the free end.

- **Conical bar suspended from its base.** At a distance x from the apex the weight below is that of a cone, $N(x) = \rho g A(x) x/3$, so the area cancels again and

  $$\delta = \int_0^L \frac{\rho g x}{3E}\, dx = \frac{\rho g L^2}{6E}$$

  one third of the uniform-bar value, because most of the material sits near the support where it carries little of the weight below it.

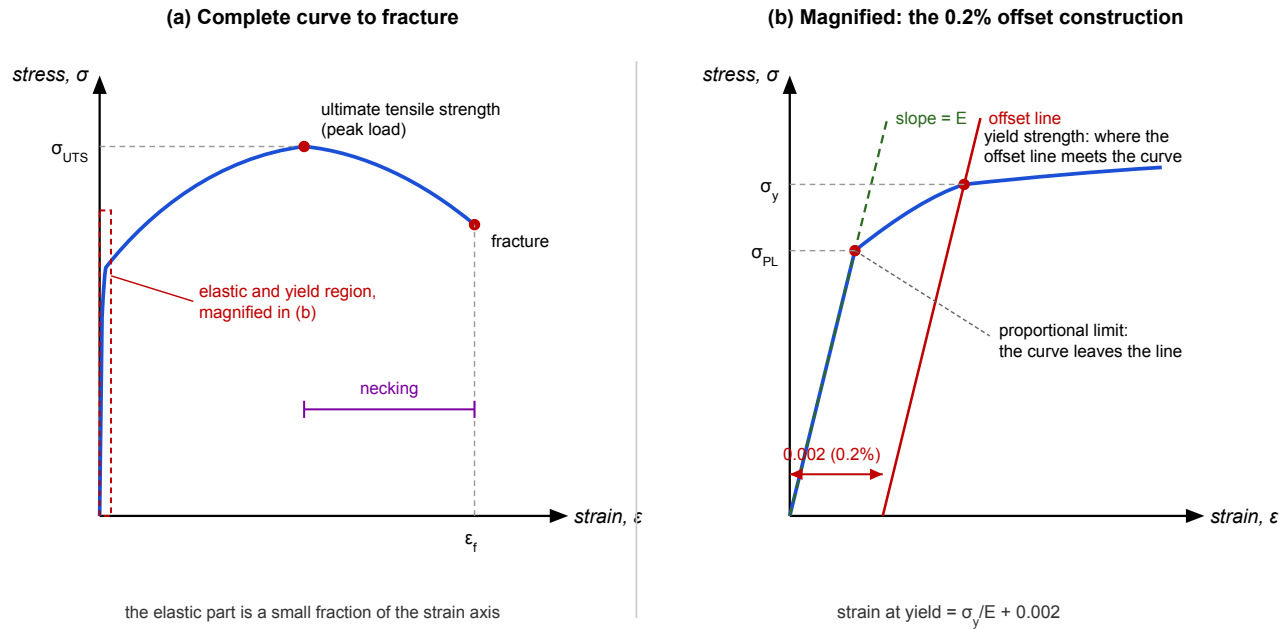
Summary of Varying-Section Results

Member	$A(x)$ or $N(x)$	Extension	Maximum stress
Uniform bar, end load W	A constant	$\delta = \dfrac{WL}{AE}$	W/A
Circular taper $D_1 \to D_2$	$A \propto (D_1 - kx)^2$	$\delta = \dfrac{4WL}{\pi E D_1 D_2}$	$\dfrac{4W}{\pi D_2^2}$
Flat taper $b_1 \to b_2$, thickness t	A linear in x	$\delta = \dfrac{WL}{Et(b_1 - b_2)} \ln \dfrac{b_1}{b_2}$	$\dfrac{W}{t\, b_2}$
Stepped bar	piecewise constant	$\delta = \sum \dfrac{N_i L_i}{A_i E_i}$	$\max(N_i/A_i)$
Uniform bar, self-weight	$N = \rho g A x$	$\delta = \dfrac{\rho g L^2}{2E}$	$\rho g L$ (at the support)
Conical bar, self-weight	$N = \rho g A(x) x/3$	$\delta = \dfrac{\rho g L^2}{6E}$	$\rho g L/3$ (at the base)

Equivalent uniform section: $d_{eq} = \sqrt{D_1 D_2}$ for a circular taper, $b_{eq} = (b_1 - b_2)/\ln(b_1/b_2)$ for a flat taper.

The Stress-Strain Curve

A tensile test pulls a standard coupon to fracture while recording force and extension. The resulting curve divides material behaviour into regions:



The two panels are the same curve at different scales, and the scale change is the point: the elastic behaviour that governs every deflection calculation in this lesson occupies a small fraction of the strain axis, so the 0.2% offset construction can only be drawn in the magnified view. The axes are deliberately unnumbered because the shape is common to ductile metals while the values are not; the figures for one particular steel are worked out in Application 3 below.

Key Points on the Curve

- **Proportional limit:** the highest stress at which stress and strain remain strictly proportional (the curve is still a straight line)

- **Yield strength (0.2% offset):** found by drawing a line with slope E starting at $\epsilon = 0.002$; where it intersects the curve is the yield point
- **Ultimate tensile strength (UTS):** the peak stress the material ever carries
- **Fracture point:** where the coupon breaks; the strain here gives the percent elongation (a ductility measure)

> **Strength vs Stiffness**
>
> Young's modulus E is the slope of the elastic line: it describes stiffness, how much force per unit strain. Yield strength and UTS describe strength, how much stress the material can carry. A soft rubber has a low E but high elongation; a cast iron has a high E but fractures with almost no plastic stretch. These are separate properties and must be checked separately.

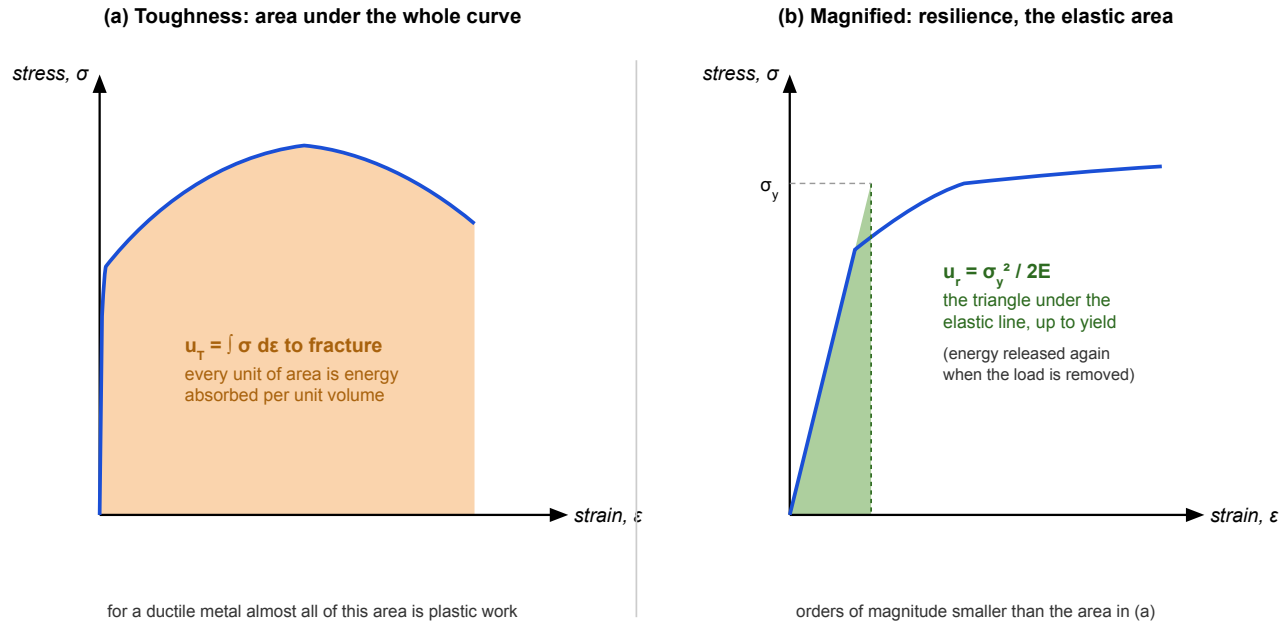
Modulus of Resilience and Modulus of Toughness

Energy Absorption Measures

$$u_r = \frac{\sigma_y^2}{2E} \quad (\text{modulus of resilience, } \mathrm{J/m^3})$$

$$u_T \approx \text{area under the full stress-strain curve} \quad (\text{modulus of toughness, } \mathrm{J/m^3})$$

Resilience measures how much elastic energy the material stores per unit volume before yielding. Toughness measures the total energy per unit volume absorbed before fracture, including all the plastic work.



Both quantities are areas under the same curve, which is why they are so easily confused, and the figure shows how different those areas are. For a ductile metal the elastic triangle is smaller than the total area by orders of magnitude, so the toughness is essentially all plastic work. Steps 2 and 3 of Application 3 evaluate both areas for a particular steel.

Shear Modulus

Shear Modulus and Its Relation to E and Poisson's Ratio

$$G = \frac{E}{2(1+\nu)}$$

$$\tau = G\gamma, \qquad \gamma = \frac{\tau}{G}, \qquad \delta_{shear} = \gamma h$$

Where:

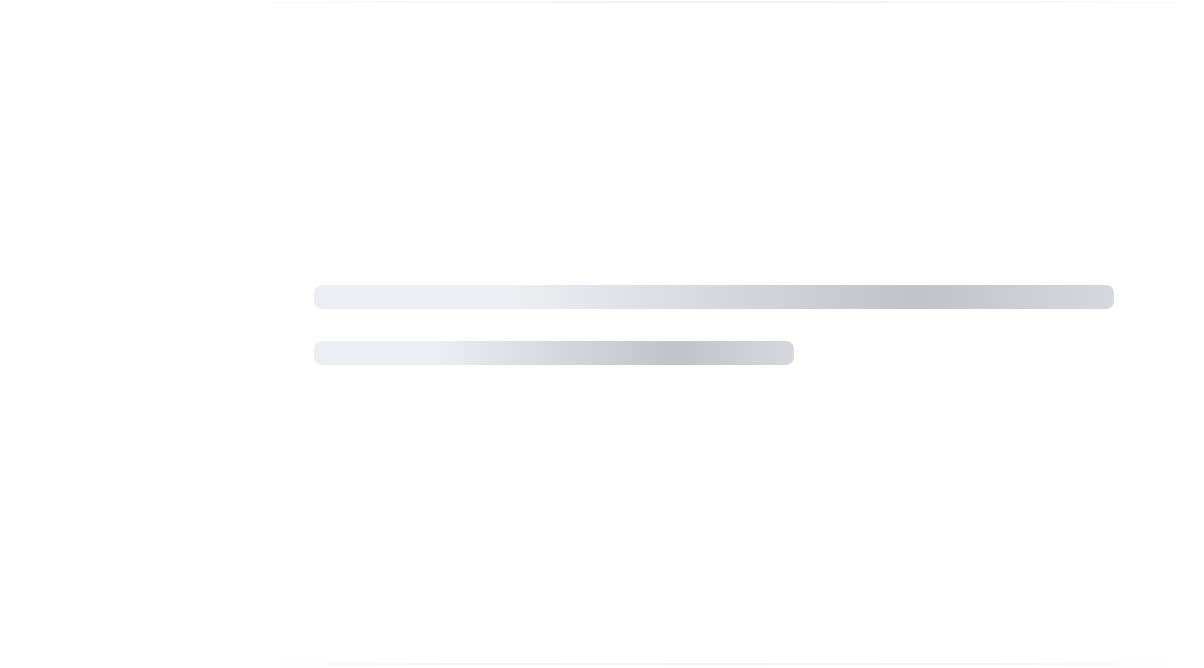
- G = shear modulus (Pa or MPa)
- τ = shear stress (Pa or MPa)
- γ = shear strain (dimensionless, in radians)
- h = thickness of the sheared layer

For metals, G falls between $0.38E$ and $0.40E$. For elastomers with $\nu \approx 0.49$, $G \approx E/2.98$, much smaller than for metals.

Material Selection Insight

A higher E means a stiffer material that deforms less under the same load. A lower E means more flexibility, which is useful for vibration damping in mounts and couplings. Steel ($E \approx 200$ GPa) is about 2.9 times stiffer than aluminum ($E \approx 70$ GPa) for the same shape, but an elastomer ($E \approx 4$ MPa) is roughly 50,000 times less stiff than steel.

Application 1: CNC Actuator Shaft, Steel vs Aluminum



The CNC Z-axis shaft must transmit a maximum axial force of 5,000 N while keeping elongation within 0.05 mm over its 400 mm length. Both a strength check and a stiffness check are required.

System Problem Statement

- **Component:** CNC Z-axis actuator shaft, circular cross-section
- **Maximum axial force:** $F = 5{,}000$ N (tension)
- **Shaft length:** $L = 400$ mm
- **Deformation limit:** $\delta \leq 0.05$ mm
- **Safety factor:** SF = 3
- **Material Option A:** Aluminum 6061-T6, $E = 70$ GPa, $\sigma_{yield} = 270$ MPa
- **Material Option B:** Steel 1045, $E = 200$ GPa, $\sigma_{yield} = 530$ MPa
- **Task:** find the minimum shaft diameter from the strength criterion and then from the stiffness criterion for each material; identify which criterion governs

Key Question: Should this shaft be aluminum or steel, and what diameter is required to meet both strength and stiffness requirements?

Run the steel and the aluminum shaft as the A and B states in the simulator and compare the deflection directly:

Open the Axial Loading Simulator

Step 1: Minimum Diameter from the Strength Criterion

> **Click to reveal the strength calculations**

Step 2: Minimum Diameter from the Stiffness Criterion

> **Click to reveal the stiffness calculations**

Step 3: Verify the Final Design and Compare Materials

> **Click to reveal the design comparison**

> **Engineering Insight**
>
> In precision positioning systems, the stiffness criterion almost always sets the shaft size, not the strength criterion. A shaft that is well within the strength limit can still fail the job by deflecting the tool out of tolerance. Both criteria must be checked, and the larger of the two required diameters governs the design.

Application 2: Tapered Steel Pull-Rod on the Machine Spindle



The shaft in Application 1 was uniform because it runs in bearings along its whole length. A drawbar or pull-rod is different: it is loaded in tension at one end and anchored at the other, nothing slides on it, and it moves with the axis, so every kilogram of it costs acceleration. That is exactly the case for a taper: put the metal where the stress is, and remove it where the stress is low.

System Problem Statement

- **Component:** steel pull-rod, circular section, tapering linearly
- **Diameter at the fixed (anchor) end:** $D_1 = 50$ mm
- **Diameter at the loaded end:** $D_2 = 25$ mm
- **Length:** $L = 900$ mm
- **Axial tensile load:** $W = 25$ kN
- **Material:** Steel 1045, $E = 200$ GPa, $\sigma_{yield} = 530$ MPa
- **Task:** find the maximum stress and where it occurs, the total extension, the stress at mid-length, and the diameter of the uniform rod that would be equally stiff; then compare mass with that uniform rod

Key Question: How much does a rod that changes section along its length stretch, and how much material does the taper save compared with the uniform rod of the same stiffness?

Step 1: Maximum Stress and Where It Acts

> **Click to reveal the maximum stress calculation**

Step 2: Total Extension of the Tapered Rod

> **Click to reveal the extension calculation**

Step 3: Stress at Mid-Length

> **Click to reveal the mid-length stress**

Step 4: Equivalent Uniform Rod and the Mass Saved

> **Click to reveal the equivalence and mass comparison**

Variation: The Same Method on a Flat Tapered Tie Plate

> **Click to reveal the flat-taper worked example**

Practice: Four Variations to Work Yourself

> **Click to reveal the practice problems and their answers**

Engineering Insight

A varying section changes nothing conceptually: the extension is still built from $d\delta = N\,dx/(AE)$, and the maximum stress is still force over the smallest area. What changes is that the two answers now come from different places in the member. The stress check is local (at the small end) and needs no calculus; the stiffness check is global (an integral over the whole length) and cannot be shortcut with an average area. Getting a taper problem right is mostly a matter of writing $A(x)$ correctly before integrating.

Application 3: Interpreting a Tensile Test of a Steel Coupon

A tensile test is the standard way to measure the mechanical properties of a material before it goes into a design. Understanding how to read the stress-strain curve lets you extract the numbers that feed every other analysis.

System Problem Statement

- **Component:** standard round steel coupon (ASTM E8)
- **Gauge length:** $L_0 = 50$ mm
- **Diameter:** $d = 12.5$ mm, giving $A_0 = \pi(12.5)^2/4 = 122.7$ mm²
- **Material:** medium-carbon steel, $E = 200$ GPa
- **Curve data (read from the test record):**
  - Proportional limit: approximately 280 MPa
  - 0.2% offset yield strength: $\sigma_y = 350$ MPa
  - Ultimate tensile strength: $\sigma_{UTS} = 520$ MPa
  - Fracture stress: 410 MPa (after necking)
  - Elongation at fracture: 22% of gauge length
- **Task:** confirm the 0.2% offset construction, compute modulus of resilience, estimate modulus of toughness, and compute percent elongation

Key Question: What are the energy-absorption characteristics of this steel, and how ductile is it?

Step 1: Confirm the 0.2% Offset Yield Construction

> **Click to reveal the 0.2% offset construction**

Step 2: Modulus of Resilience

> **Click to reveal the resilience calculation**

Step 3: Estimate the Modulus of Toughness

> **Click to reveal the toughness estimate**

Step 4: Percent Elongation (Ductility)

> **Click to reveal the percent elongation**

Engineering Insight

A stress-strain curve tells you four different things about a material: its stiffness (the slope E), its yield strength (where permanent deformation begins), its ultimate strength (the peak load it can ever carry), and its ductility (how much it deforms before breaking). All four numbers are needed for a complete design check, and none of them can substitute for another.

Application 4: Shear Modulus and a Bonded Elastomer Anti-Vibration Mount

Shear modulus governs any component that is loaded across its thickness rather than along its length. Bonded elastomer mounts are a common example: the rubber layer is

glued between two metal plates, and when the structure moves horizontally the rubber is sheared. The mount's compliance in shear is exactly what provides the vibration isolation.

System Problem Statement

- **Component:** bonded natural-rubber anti-vibration mount
- **Bonded pad dimensions:** 80 mm x 80 mm (shear area $A = 6{,}400$ mm²)
- **Rubber layer thickness:** $h = 20$ mm
- **Material:** natural rubber, $E = 4$ MPa, $\nu = 0.49$
- **Applied horizontal shear force:** $F = 800$ N
- **Task:** compute G from E and ν, then find shear stress τ, shear strain γ, and the lateral shear deflection δ_{shear}

Key Question: By how much does the mount deflect laterally under an 800 N horizontal load, and is that deflection in the expected design range for vibration isolation?

Step 1: Shear Modulus from E and Poisson's Ratio

> **Click to reveal the shear modulus calculation**

Step 2: Shear Stress in the Rubber Layer

> **Click to reveal the shear stress calculation**

Step 3: Shear Strain and Lateral Deflection

> **Click to reveal the shear strain and deflection**

Engineering Insight

The shear modulus is not an independent material property: it follows directly from Young's modulus and Poisson's ratio through $G = E/(2(1+\nu))$. For metals, $G \approx 0.38E$; for rubbers, $G \approx E/3$. Whenever a component is loaded in shear (a lap joint, an elastomer mount, a keyed shaft), the calculation uses $\tau = F/A$ and $\gamma = \tau/G$, in exactly the same pattern as the axial case but with the shear quantities throughout.

Design Guidelines for Strain and Material Properties

Check stiffness and strength separately

In precision systems, the deformation limit almost always sets a larger minimum cross-section than the strength criterion. Run both calculations and take the larger diameter.

Read the full stress-strain curve

Yield strength and UTS are on the same curve, but they mean different things. Yield is where permanent deformation begins; UTS is the peak load. Using UTS for a yield-based design check is unsafe.

Know G for shear-loaded parts

Any component carrying shear (a pin, an adhesive joint, a rubber mount, a splined coupling) requires the shear modulus, not Young's modulus. Compute it from $G = E/(2(1+\nu))$ if you only have E and ν.

Ductility provides failure warning

A material with more than 5% elongation at fracture deforms visibly before it breaks. This gives warning time. Brittle materials fracture suddenly, with no prior deformation, which is why ductility matters in safety-critical parts.

Instrument it

The stiffness criterion that sized the actuator shaft is something you can measure directly. Mount an LVDT or a dial indicator at the loaded end and read the real deflection under a known load; its slope is the actual axial stiffness, which is rarely exactly what the handbook modulus predicts (fits, surface condition, and bearing compliance all add compliance). An ESP32 or STM32 logging the LVDT output lets you trend the measured stiffness on siliconwit.io and verify the shaft meets the tolerance before you trust the part in production.

Summary and Next Steps

Key Concepts Mastered

1. **Axial deflection** follows $\delta = FL/(AE)$. In precision systems the stiffness criterion (deformation limit) almost always sets a larger minimum section than the strength criterion, and both must be checked.
2. **Non-uniform members** are all handled by one relation, $\delta = \int_0^L N(x)\,dx/(A(x)E)$: a linear circular taper gives $\delta = 4WL/(\pi E D_1 D_2)$, a flat taper gives $\delta = WL\ln(b_1/b_2)/(Et(b_1 - b_2))$, a stepped bar gives $\sum N_i L_i/(A_i E_i)$, and self-weight gives $\rho g L^2/(2E)$ for a uniform bar. The maximum stress is always N/A_{min} at the smallest section and needs no integration; the extension can never be found from an average area.
3. **The tensile test curve** captures stiffness (E, the slope), yield strength (0.2% offset construction), UTS (peak), fracture strain (ductility), and the energy per unit volume the material can absorb (resilience and toughness).
4. **Shear modulus** is related to Young's modulus and Poisson's ratio by $G = E/(2(1+\nu))$. Shear-loaded parts (elastomer mounts, pins, adhesive joints) use $\tau = F/A$ and $\gamma = \tau/G$ to find the shear deflection $\delta_{shear} = \gamma h$.

Results at a Glance

Application	Key result	Governing criterion
CNC shaft: aluminum 27 mm, steel 16 mm	Al: $\delta = 0.050$ mm, mass 0.62 kg; steel: $\delta = 0.050$ mm, mass 0.63 kg	Stiffness governs both
Tapered pull-rod (50 mm → 25 mm, 900 mm, 25 kN)	$\sigma_{max} = 50.9$ MPa at the small end, $\delta = 0.115$ mm, $\sigma_{mid} = 22.6$ MPa, $d_{eq} = 35.4$ mm, 41.7% less material	Stiffness governs (SF = 10.4 on strength)
Steel tensile coupon	$u_r = 306$ kJ/m³, $u_T \approx 95.3$ MJ/m³, ductility 22%	Yield at 350 MPa, UTS 520 MPa
Elastomer mount (80 mm x 80 mm, 20 mm thick)	$G = 1.34$ MPa, $\tau = 0.125$ MPa, $\gamma = 0.093$ rad, $\delta_{shear} = 1.86$ mm	Shear governs deflection

A Note on Tools

Every number in this lesson came from four relations ($\delta = FL/(AE)$, its non-uniform form $\delta = \int N\,dx/(AE)$, $u_r = \sigma_y^2/(2E)$, and $G = E/(2(1+\nu))$) and a calculator. Finite-element software is not needed for axial or shear members, tapered ones included; the hand calculation is the design, and simulation is for validating complex geometry after the sizing is done.

Next, Compound Bars and Composite Systems extends the axial analysis to members made of two different materials in series or parallel, which is the situation in every bolted joint, press fit, and bi-material actuator.

Comments

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