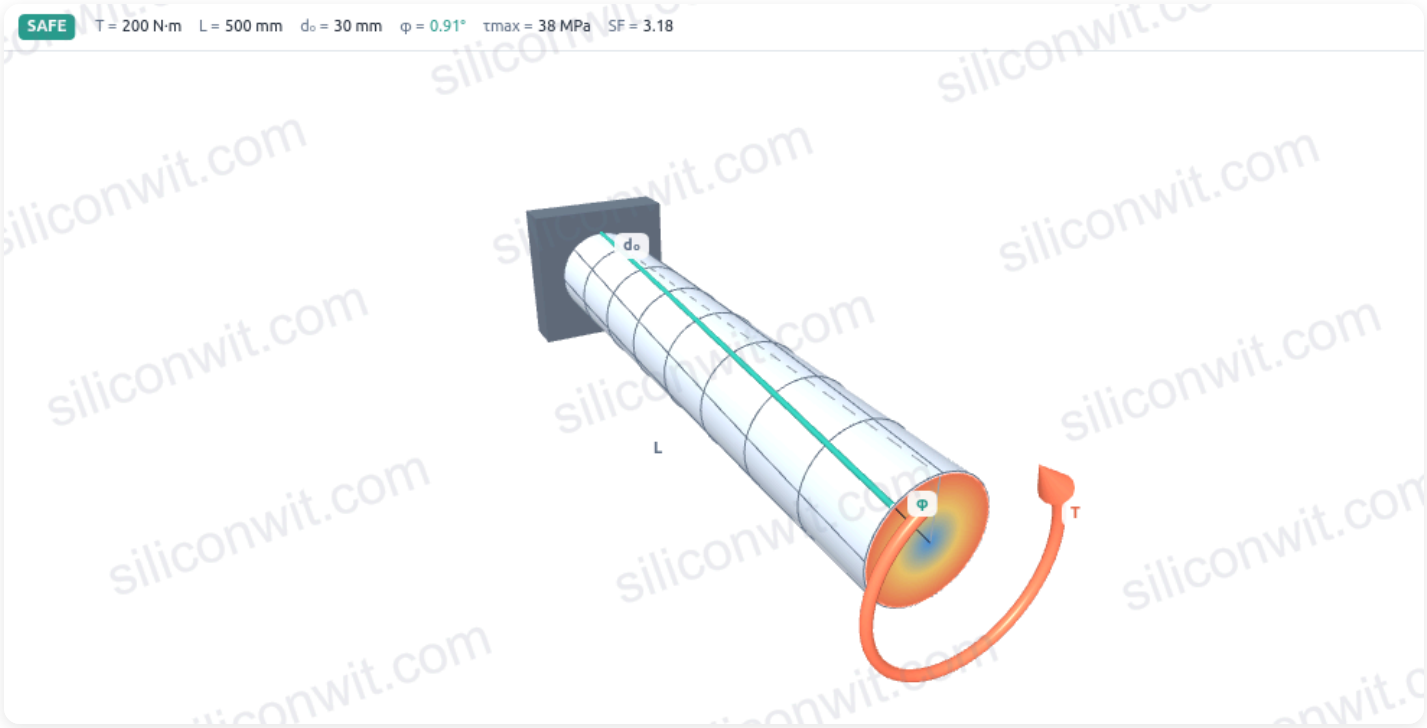
Lesson 1.5: Torsion of Circular Shafts



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A shaft that transmits torque without breaking can still fail functionally if it twists too much: the angular deflection throws off the timing in a Geneva indexing mechanism, causes vibration in a coupling, or misaligns a sensor mounting. Designing against torsional failure means checking both the shear stress (will it yield?) and the angle of twist (will it stay accurate?). In this lesson you will analyze three real cases, a Geneva mechanism crankshaft, a power-transmitting motor shaft, and a hollow-versus-solid

shaft comparison, learning the torsion formula, polar moment of inertia, and the design trade-offs that keep rotating shafts strong and stiff. #Torsion #ShearStress #ShaftDesign

Learning Objectives

By the end of this lesson, you will be able to:

1. **Derive** the torsional shear stress formula and explain why stress peaks at the outer surface
2. **Calculate** peak shear stress and angle of twist for solid and hollow circular shafts
3. **Convert** power and rotational speed into torque, and apply the result to a shaft design
4. **Compare** a hollow and a solid shaft carrying the same torque, and quantify the material saving

> **Try it as you read**
>
> This lesson pairs with the Shaft Torsion Simulator: twist a shaft in 3D and read the shear stress across the radius and the angle of twist along the length, then compare a hollow section against a solid one. The Shaft Torsion Experiments reproduce the worked examples step by step with Python.

Real-World System Problem: Geneva Mechanism Crankshaft

The Geneva mechanism converts continuous rotation into intermittent indexing motion. It appears in film-transport mechanisms, rotary indexing tables, and automated assembly equipment. The drive crankshaft spins continuously while the Geneva wheel advances one slot at a time, and every engagement event sends a torque spike through the crankshaft.

The Torsional Challenge

> **Engineering Question:** How do we size the crankshaft so it can transmit peak torque without yielding, and keep the angular twist small enough to preserve timing accuracy?

Both criteria matter. A shaft that survives the stress but winds up by several degrees will cause the Geneva wheel to over-travel or under-travel its slot, jamming the mechanism or marking parts at the wrong position.

Why Torsional Analysis Matters

⚠ Preventing shaft failure

Peak shear stress compared against the allowable value tells you whether the shaft survives the worst engagement torque before it is ever assembled.

⚙ Preserving timing accuracy

The angle of twist formula sets how stiff a shaft must be so that angular deflection stays within the positioning tolerance of the driven mechanism.

Choosing solid versus hollow

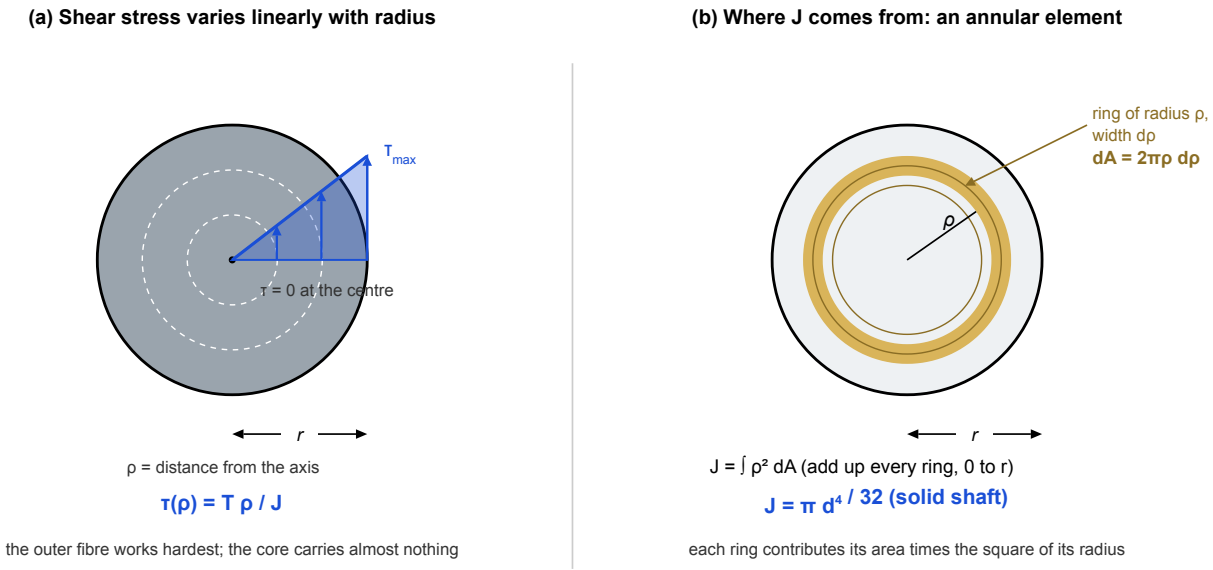
Most of the torsional stiffness in a circular shaft comes from material near the outer surface, so removing the low-stress core saves weight with little penalty in strength.

Sizing for power transmission

Motors and gearboxes are specified by power and speed, not torque. Converting P and n to T is the first step in every shaft design.

Fundamental Theory: Torsional Mechanics

When a circular shaft carries a twisting moment (torque), every cross-section resists by developing shear stress that varies linearly from zero at the centre to a maximum at the outer surface. Three formulas govern everything in this lesson.



Panel (b) is where $J = \pi d^4/32$ comes from: each thin ring contributes its own area weighted by the *square* of its radius, so material near the surface counts for far more than material near the axis. That single fact explains both the stress distribution in panel (a) and why hollow shafts are efficient.

Torsional shear stress

$$\tau = \frac{T\,r}{J}$$

Where:

- τ = shear stress (Pa)
- T = applied torque (N·m)
- r = radial distance from the shaft centre (m); maximum stress occurs at $r = d/2$
- J = polar second moment of area (m^4)

The stress is highest at the outer fibre because r is largest there. The centre carries no shear stress at all, which is why hollow shafts waste so little capacity.

Polar second moment of area

For a solid shaft of diameter d:

$$J_{solid} = \frac{\pi d^4}{32}$$

For a hollow shaft with outer diameter d_o and inner diameter d_i:

$$J_{hollow} = \frac{\pi (d_o^4 - d_i^4)}{32}$$

J captures how the cross-section's area is distributed about the axis. Because diameter enters as the fourth power, a modest increase in diameter raises J dramatically.

Angle of twist

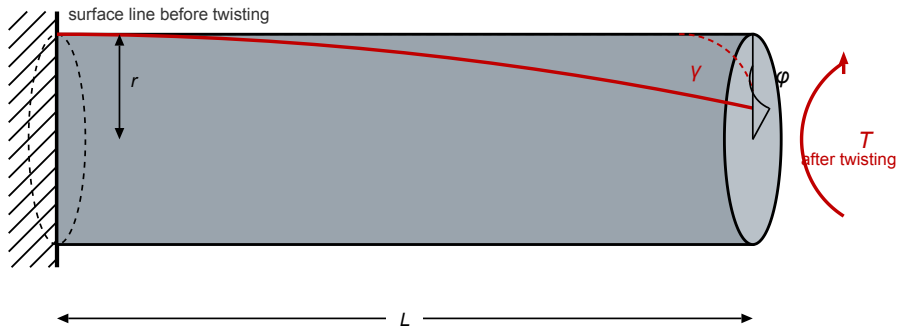
$$\varphi = \frac{T L}{G J}$$

Where:

- φ = angle of twist (radians)
- L = shaft length (m)
- G = shear modulus of the material (Pa); for steel $G \approx 79$ GPa

Angular deflection grows with torque and length, and shrinks with material stiffness and section stiffness. A shaft that passes the stress check can still fail the stiffness check if it is long or lightly proportioned.

Angle of twist: the geometry behind φ = T L / (G J)



The same arc at the surface, measured two ways:

arc = r φ (rotation of the end) = L γ (shear of the surface)

γ = r φ / L → τ = G γ = G r φ / L

equate with τ = T r / J → φ = T L / (G J)

The formula is not arbitrary: a straight line scribed on the surface becomes a helix, and the arc swept at the free end can be measured two ways, as $r\varphi$ from the rotation or as $L\gamma$ from the shear. Equating them gives $\gamma = r\varphi/L$, and combining that with $\tau = G\gamma$ and $\tau = Tr/J$ produces $\varphi = TL/(GJ)$.

Power and torque

A motor or gearbox rated at power P (watts) running at angular speed ω (rad/s) delivers:

$$T = \frac{P}{\omega}, \qquad \omega = \frac{2\pi\, n}{60}\ (\text{rad/s, with } n \text{ in rpm})$$

This is the gateway calculation for every driven shaft: convert the nameplate data to torque before applying the torsion formula.

> **Engineering Units**
>
> Working in SI throughout keeps the units consistent. Torque in N·m, length in m, G in Pa, and J in m_4 gives φ in radians and τ in Pa. Convert Pa to MPa by dividing by 10^6.
>
> The shortcut 1 MPa = 1 N/mm^2 lets you use mm throughout: convert d to m only for J (or keep mm and divide by 10^{12} to get m_4). The clearest approach is to stay in SI metres and convert the final answer.

Application 1: Geneva Mechanism Crankshaft

The Geneva drive crankshaft below is the original motivating problem. Three design questions are answered in sequence: is the 25 mm shaft strong enough? if not, what diameter is needed for strength? and once resized, does it meet the timing requirement?

System Problem Statement

- **Component:** Geneva mechanism drive crankshaft, solid steel (G = 80 GPa, τ_{allow} = 180 MPa)
- **Shaft diameter:** 25 mm, **Length:** 200 mm
- **Continuous torque:** T_{cont} = 50 N·m (normal running)
- **Peak engagement torque:** T_{peak} = 150 N·m (slot entry)
- **Required safety factor:** 3.0 on peak torque
- **Precision requirement:** angular twist must stay within $\pm 0.5^\circ$

Key Question: Can this 25 mm crankshaft safely handle 150 N·m peak torque with SF = 3.0, and will the angular twist stay within the $\pm 0.5^\circ$ precision budget?

Step 1: Polar Moment of Inertia for the 25 mm Shaft

> **Click to reveal the polar moment calculation**

Step 2: Torsional Stress Check

> **Click to reveal the torsional stress analysis**

Step 3: Angle of Twist Check

> **Click to reveal the angular deformation analysis**

Engineering Insight

The 25 mm crankshaft is strong enough (SF = 3.7) but marginally too flexible (twist = 0.56°, just over the 0.5° limit). The governing design criterion here is stiffness, not strength, which is typical for precision indexing mechanisms. Increasing the diameter by 1 mm resolves both and adds very little mass.

Application 2: Sizing a Power-Transmitting Motor Shaft

A small conveyor drive unit has a 5 kW induction motor running at 960 rpm. The output shaft connects the motor to a worm gearbox over a span of 400 mm. The shaft material is steel with $G = 79$ GPa and an allowable shear stress of 60 MPa. A 20 mm solid shaft has been proposed. Verify the shear stress and angle of twist.

System Problem Statement

- **Component:** solid steel motor output shaft, d = 20 mm, L = 400 mm
- **Motor power:** P = 5 kW at n = 960 rpm
- **Material:** steel, G = 79 GPa, τ_{allow} = 60 MPa
- **Task:** compute the delivered torque, the peak shear stress, and the angle of twist; check the stress against the allowable

Key Question: Does the 20 mm shaft keep shear stress within the 60 MPa allowable, and how many degrees does it twist over its 400 mm length?

Enter the motor shaft in the simulator and read the peak shear stress and the angle of twist over its length:

Open the Shaft Torsion Simulator

Step 1: Torque from Power and Speed

> **Click to reveal the torque calculation**

Step 2: Polar Moment and Peak Shear Stress

> **Click to reveal the stress calculation**

Step 3: Angle of Twist

> **Click to reveal the angle of twist calculation**

Engineering Insight

Starting from nameplate data (power and rpm), the torque follows from $T = P/\omega$, and the rest of the shaft analysis proceeds by the same formulas as any other torsion problem. The stress of 31.7 MPa leaves a factor of 1.89 against the allowable, meaning the proposed 20 mm shaft is adequate for steady drive duty but has limited margin for shock or fatigue. Because shear stress scales as $1/d^3$, the safety factor scales as d^3: increasing to 22 mm or 25 mm would raise SF to about 2.5 or 3.7 respectively at negligible cost.

Application 3: Hollow Versus Solid Shaft for the Same Torque

A drive shaft must transmit T = 200 N·m. The allowable shear stress is 60 MPa. Two designs are compared: a minimum solid shaft sized to just meet the allowable, and a hollow shaft with the same outer diameter as the next standard size up. The goal is to show how much cross-sectional area (and therefore material) the hollow design saves.

System Problem Statement

- **Torque:** T = 200 N·m
- **Allowable shear stress:** τ_{allow} = 60 MPa
- **Design A:** solid shaft, diameter sized to meet the allowable
- **Design B:** hollow shaft, outer diameter d_o = 32 mm, inner diameter d_i = 26 mm
- **Task:** compute J, peak shear stress, and cross-sectional area for each; find the area saving

Key Question: How much lighter is the hollow shaft compared to the solid shaft for the same torque and the same allowable stress?

Step 1: Size the Solid Shaft

> **Click to reveal the solid shaft sizing**

Step 2: Analyse the Hollow Shaft

> **Click to reveal the hollow shaft analysis**

Step 3: Compare and Quantify the Saving

> **Click to reveal the comparison**

Engineering Insight

The material near the centre of a solid shaft contributes very little to J because the lever-arm r is tiny there, yet it adds full weight. Removing that core with a hollow design here cuts mass by nearly half while keeping peak stress slightly below the solid shaft's value. The trade-off is a larger outer diameter (32 mm vs 26 mm) and a slightly more complex machining or forming process. In weight-sensitive applications such as aerospace linkages, robot arms, and high-speed spindles, that trade is almost always worth making.

Design Guidelines for Torsional Loading

Convert power to torque first

Every motor-driven shaft starts with $T = P/\omega$. Get the torque right before applying any geometry formula; a factor-of-ten error in torque flows straight through to the stress.

Check strength and stiffness separately

A shaft can pass the shear-stress check and still fail if the angle of twist throws off a timing mechanism or misaligns a coupling. Both criteria must be satisfied.

Increase diameter over length

Diameter enters J as the fourth power and enters the stress as the third power, so increasing d by 10% reduces stress by 25% and increases stiffness by 46%. Length reduction offers far less return.

Consider hollow sections for weight

The centre of a shaft contributes little to torsional stiffness. For large diameters or weight-sensitive designs, hollow sections routinely save 40 to 60% of cross-sectional area with only a modest increase in outer size.

Instrument it

The two quantities this lesson sizes, shear stress and angle of twist, are both measurable in a running system. A strain-gauge bridge bonded at 45 degrees to the shaft axis reads the principal strains from torsion; from those, the shear stress and torque follow directly. An optical or magnetic encoder pair, one at each end of the shaft, reads the actual twist in service. An ESP32 or STM32 reading both sensors can stream torque and twist data to siliconwit.io and flag when a shaft is winding up past its precision limit or approaching its fatigue threshold.

Summary and Next Steps

Key Concepts Mastered

1. **Torsional shear stress** is $\tau = Tr/J$, peaks at the outer surface, and is zero at the centre; the formula holds for any circular shaft, solid or hollow.
2. **Polar moment** $J = \pi d^4/32$ for a solid shaft and $J = \pi(d_o^4 - d_i^4)/32$ for hollow; diameter enters as the fourth power, making it the most effective design variable.
3. **Angle of twist** is $\varphi = TL/(GJ)$; stiffness governs precision mechanisms just as strength governs high-stress shafts, and both checks are mandatory.
4. **Power conversion** $T = P/\omega$ is the gateway to every motor-shaft calculation; always carry the full torque value into the torsion formula.
5. **Hollow shafts** remove the low-stress core, saving roughly half the cross-sectional area for a modest increase in outer diameter, as shown in Application 3.

Results at a Glance

Application	Key inputs	Peak τ (MPa)	Angle of twist	Notes
Geneva crankshaft	T = 150 N·m, d = 25 mm	48.9	0.56°	Strength OK; stiffness marginally exceeded; use 26 mm
Motor shaft	P = 5 kW, 960 rpm, d = 20 mm	31.7	0.92° over 400 mm	SF = 1.89; adequate for steady drive
Solid shaft	T = 200 N·m, d = 26 mm	58.0		A = 530.9 mm_2
Hollow shaft	T = 200 N·m, d_o/d_i = 32/26 mm	55.1		A = 273.3 mm_2; 48.5% lighter

A Note on Tools

Every result in this lesson came from four formulas ($\tau = Tr/J$, $J = \pi d^4/32$, $\varphi = TL/(GJ)$, $T = P/\omega$) and a calculator. No finite-element package is needed for circular shafts under uniform torque; the hand calculation is the design, and simulation only confirms it for complex geometry such as stepped or keyway-notched shafts.

Next, Thin-Walled Pressure Vessels applies a similar two-stress approach to cylindrical shells, where internal pressure simultaneously creates both hoop (circumferential) and longitudinal normal stresses in the wall.

Comments

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