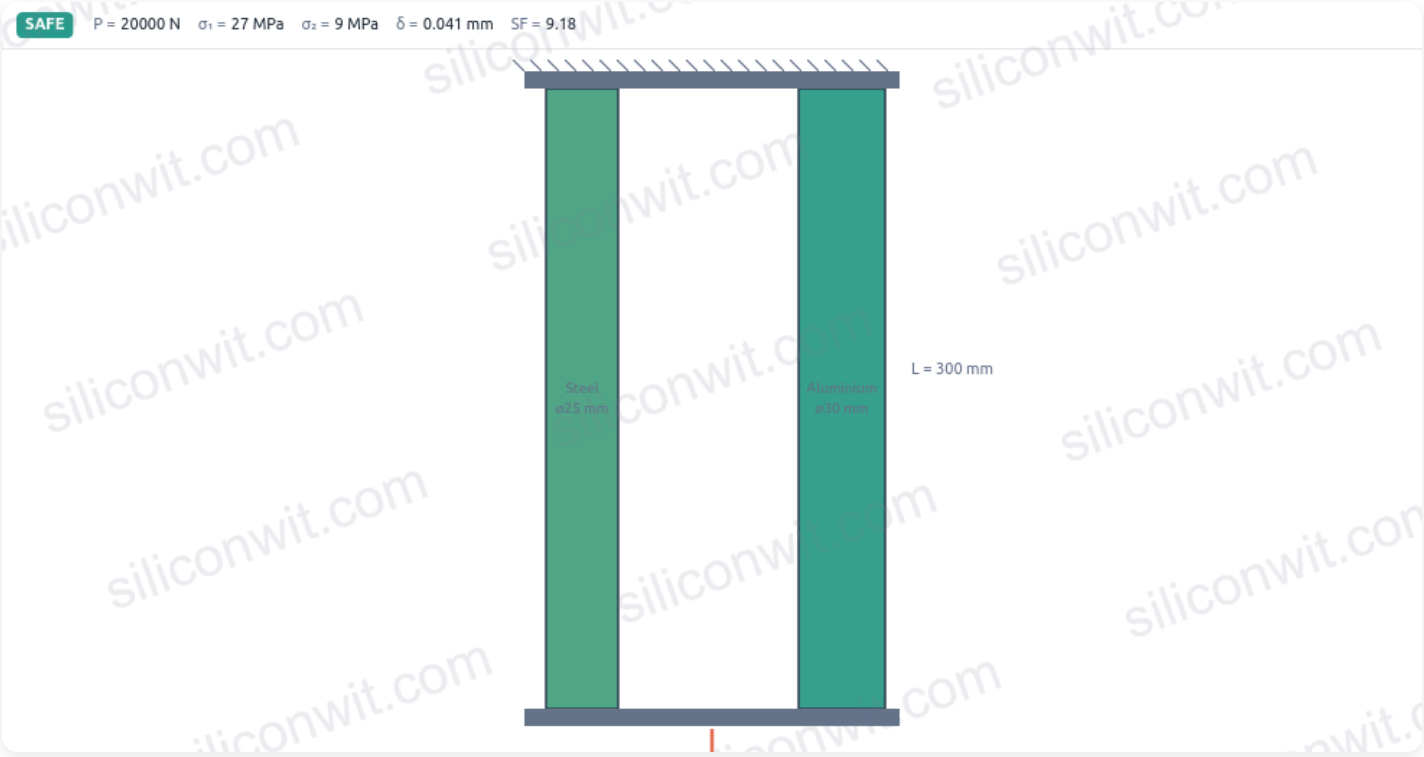
Lesson 1.3: Compound Bars and Composite Systems



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When a load passes through two materials joined end to end, steel and aluminum for example, each material carries a different stress and stretches by a different amount. Ignoring this mismatch leads to designs where one segment yields while the other is

barely loaded, or where differential expansion under temperature changes introduces unexpected forces at the joint. In this lesson you will analyze three compound systems: a linear actuator rod (series, unequal areas), a steel-core aluminum-tube column (parallel, equal shortening), and a stepped tie rod (series, total elongation as a sum). #CompoundBars #LoadSharing #MultiMaterial

Learning Objectives

By the end of this lesson, you will be able to:

1. **Distinguish** between parallel (equal-deformation) and series (equal-force) compound bar arrangements
2. **Apply** equilibrium and deformation compatibility to find the load carried by each material in a parallel system
3. **Calculate** the stress in each segment and the common deflection when materials share the same shortening
4. **Sum** segment elongations to find total deflection in a series compound bar carrying the same axial force throughout

> **Try it as you read**
>
> This lesson pairs with the Axial Loading Simulator: build a compound bar and watch the load split between the two materials by axial stiffness, not by area, exactly as the analysis below predicts. The Axial Loading Experiments reproduce the worked examples step by step with Python.

Real-World System Problem: Multi-Material Actuator and Column Design

Industrial machines combine materials deliberately: a stiff steel core to carry the bulk of a compressive load, a lighter aluminum jacket to add cross-sectional area at low cost, or a steel-aluminum stepped rod to save weight along the lightly stressed length. Each combination falls into one of two fundamental arrangements, and the analysis depends on which one it is.

The Compound Bar Problem

> **Engineering Question:** When a single axial load is applied to a system made of two or more materials, how does the load split between them, and how much does each material deform? The answer depends entirely on whether the materials deform together (parallel) or the same force passes through them in succession (series).

Why Multi-Material Analysis Matters

⚠ Preventing hidden overload

In a parallel system, the stiffer material attracts more load than its area alone would suggest. Ignoring this can leave one material near its yield limit while the other is barely stressed.

⚙ Weight and cost optimization

Using aluminum where stresses are low and steel only where they are high can cut component weight by 30 to 40 percent without compromising strength.

Deformation compatibility

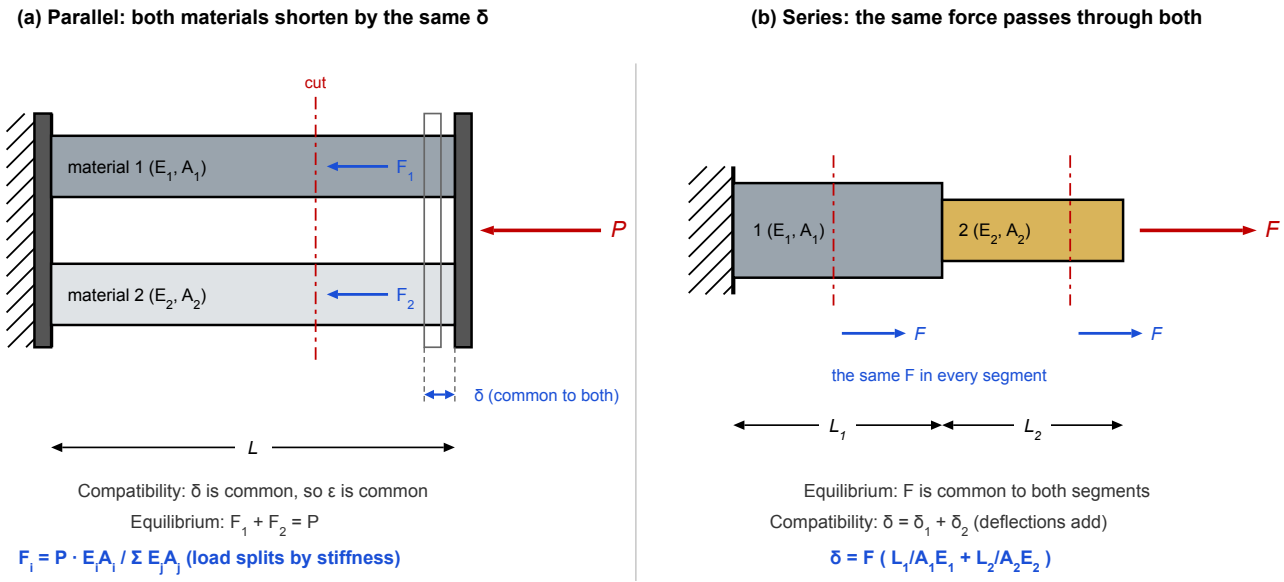
Whenever two materials are bonded or bolted together, their deformations must match at every interface. Violating this constraint is the most common error in compound bar analysis.

System-level reliability

A compound rod that yields in one material will redistribute load to the other, often catastrophically. The analysis predicts and prevents that redistribution.

Fundamental Theory: Parallel and Series Compound Bars

Every compound bar problem reduces to one of two arrangements, and getting the right one is the first step.



Read the two panels as a pair of questions: *what is shared, and what adds up?* In parallel the deformation is shared and the forces add; in series the force is shared and the deformations add. Every result below follows from that one distinction.

Case 1: Parallel Arrangement (Equal Deformation)

In a parallel compound bar the materials share the same length and the same end conditions: both shorten (or lengthen) by exactly the same amount δ. A steel rod inside an aluminum sleeve, both compressed between the same two rigid end plates, is the standard example.

Parallel: compatibility and load split

Because both materials shorten by δ over length L, they share the same strain ε:

$$\varepsilon = \frac{\delta}{L} \quad \text{(same for all materials)}$$

The stress in each material follows from Hooke's law:

$$\sigma_i = E_i \, \varepsilon$$

Equilibrium requires the loads to sum to the applied total:

$$P_{total} = \sum_i F_i = \sum_i \sigma_i A_i = \varepsilon \sum_i E_i A_i$$

Solving for the load carried by material i:

$$F_i = P_{total} \cdot \frac{E_i A_i}{\sum_j E_j A_j}$$

The load splits in proportion to the **axial stiffness** EA of each material, not its area alone.

Why the stiffer material dominates

If steel has $E = 200$ GPa and aluminum has $E = 70$ GPa, a steel section with the same area as an aluminum section carries $200/70 \approx 2.86$ times more load. This ratio only changes if you deliberately give the softer material a larger cross-sectional area to compensate.

Case 2: Series Arrangement (Equal Force)

In a series compound bar, the axial force is the same in every segment (from equilibrium of any cross-section between the applied loads). The deformations of the segments add up to give the total elongation.

Series: same force, deflections add

For each segment i with length L_i, area A_i, and modulus E_i:

$$\sigma_i = \frac{F}{A_i}$$

$$\delta_i = \frac{F L_i}{A_i E_i}$$

Total elongation:

$$\delta_{total} = \sum_i \delta_i = F \sum_i \frac{L_i}{A_i E_i}$$

Each segment has its own stress level because the areas differ, but the force is the same throughout.

Checking which case applies

Ask: are the two materials attached at both ends to the same rigid plates (or the same attachment points), so neither can shorten more than the other? If yes, it is a parallel system. If one material is simply bolted end-to-end with the other so the load passes through them in sequence, it is a series system.

Application 1: Linear Actuator Rod with Steel and Aluminum Sections



An industrial linear actuator rod is made in two sections joined end to end. The steel section is short and compact, sized for the confined end of the actuator; the aluminum section is longer and wider, reducing overall weight along the stroke. A tensile force is applied at one end and reacted at the other, so the same force passes through both sections. This is a series arrangement.

System Problem Statement

- Steel section: length $L_1 = 200$ mm, diameter $d_1 = 25$ mm, Steel 1045, $E_1 = 200$ GPa, $\sigma_{y1} = 530$ MPa
- Aluminum section: length $L_2 = 300$ mm, diameter $d_2 = 30$ mm, Aluminum 6061-T6, $E_2 = 70$ GPa, $\sigma_{y2} = 270$ MPa
- Applied tensile force: $F = 20{,}000$ N
- Required safety factor: 3.0

Key Question: What is the stress in each section, and how much does the rod stretch in total?

Step 1: Cross-Sectional Areas

> **Click to reveal the area calculations**

Step 2: Axial Stiffness and Load Distribution

> **Click to reveal the stiffness and load calculations**

Step 3: Stress and Safety Factors

> **Click to reveal the stress calculations**

Step 4: Elongation of Each Section and Total Stretch

> **Click to reveal the deflection calculation**

Engineering Insight

In a series compound rod the force is the same everywhere, but the stresses and deflections differ because the areas and moduli differ. The section with the smallest AE product stretches most. In stroke-sensitive actuators, even 0.16 mm of compliance can matter: it must be accounted for in the control loop dead-band.

Application 2: Steel Core Inside an Aluminum Tube Under Shared Compression



A structural column is built by sliding a solid steel rod through the centre of an aluminum tube and compressing both between the same two end plates. Both materials shorten by the same amount (they share the same end fixings), so the load splits by axial stiffness. This is a parallel arrangement.

System Problem Statement

- Steel solid rod: diameter $d_s = 30$ mm, $E_s = 200$ GPa, $\sigma_{ys} = 350$ MPa
- Aluminum tube: inner diameter $d_{ai} = 30$ mm, outer diameter $d_{ao} = 50$ mm, $E_a = 70$ GPa, $\sigma_{ya} = 270$ MPa
- Shared length: $L = 250$ mm
- Total compressive load applied to the end plates: $P = 120{,}000$ N

Key Question: How does the 120 kN split between the steel and the aluminum, what is the stress in each, and by how much does the column shorten?

Load a compound steel-and-aluminum bar in the simulator and watch the stiffer steel draw more than its area share of the load:

Open the Axial Loading Simulator

Step 1: Cross-Sectional Areas

> **Click to reveal the area calculations**

Step 2: Axial Stiffness of Each Material

> **Click to reveal the axial stiffness values**

Step 3: Load Carried by Each Material

> **Click to reveal the load distribution**

Step 4: Stress in Each Material and Safety Factors

> **Click to reveal the stress and safety factor calculations**

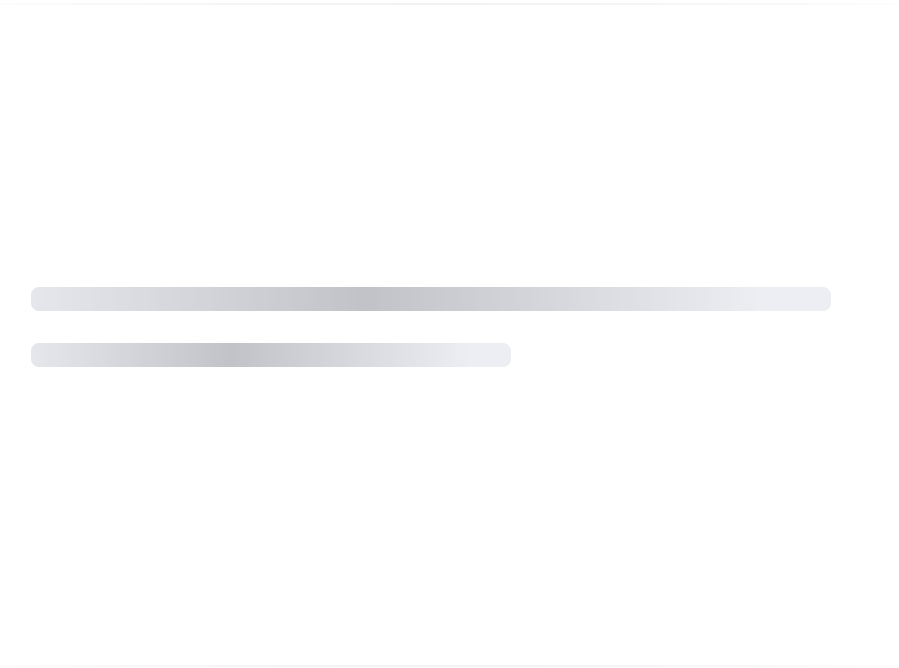
Step 5: Common Deflection

> **Click to reveal the deflection calculation**

Engineering Insight

Steel carries 62 percent of a 120 kN load through only 36 percent of the cross-sectional area because modulus multiplies area in the load-sharing rule. When you need a compact, stiff column, shrinking the aluminum and growing the steel cross-section shifts the balance further toward steel, but the governing safety factor stays at the steel end. The aluminum in this design has a generous margin and could be made thinner without reducing column safety.

Application 3: Stepped Steel-Aluminum Tie Rod in Tension



A stepped tie rod carries a single tensile force from one end to the other through two segments of different material and diameter. The force is the same in every cross-section (series arrangement), so the stresses differ because the areas differ, and the total elongation is the sum of the two segment elongations.

System Problem Statement

- Steel segment: diameter $d_s = 20$ mm, length $L_s = 300$ mm, $E_s = 200$ GPa, $\sigma_{ys} = 350$ MPa
- Aluminum segment: diameter $d_a = 30$ mm, length $L_a = 400$ mm, $E_a = 70$ GPa, $\sigma_{ya} = 270$ MPa
- Applied tensile force: $F = 30{,}000$ N throughout

Key Question: What stress does each segment carry, and what is the total elongation of the rod?

Step 1: Cross-Sectional Areas

> **Click to reveal the area calculations**

Step 2: Stress in Each Segment

> **Click to reveal the stress calculations**

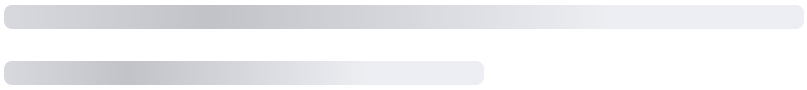
Step 3: Elongation of Each Segment and Total Elongation

> **Click to reveal the elongation calculations**

Engineering Insight

In a series rod the material with the smallest AE product stretches most per unit length; in this case the aluminum segment contributes 63 percent of the total elongation despite being 57 percent of the total length. If this rod positions a tool or a sensor, the soft segment dominates the compliance budget. One practical fix is to increase the aluminum diameter until its AE matches the steel segment's, which balances the stretch contribution between the two materials.

Design Guidelines for Compound Systems



Identify the arrangement first

Before writing any equation, decide whether the materials deform together (parallel) or carry the same force in sequence (series). The governing equations are completely different for the two cases.

Load splits by EA in parallel

In a parallel system, use $F_i = P \cdot EA_i / \sum EA_j$. The stiffer material always attracts more load than its area fraction alone would imply, sometimes far more.

Check every material separately

Each segment has its own stress and its own safety factor against yield. Passing one does not guarantee the other passes. In most compound systems the materials have different yield strengths, so both checks are needed.

Sum deflections in series, share in parallel

Total elongation in a series bar is the sum of segment elongations. In a parallel bar, one common deflection closes the problem. Confusing the two is the source of most compound-bar errors.

Instrument it

Load sharing in a parallel system is invisible until you measure it. Bond a strain gauge to each material; because they share the same deflection but differ in stiffness, the two strain readings reveal how the load actually splits, usually far from what area fractions alone would suggest. Read both gauges with an ESP32 or STM32 and you can watch the split live on siliconwit.io and confirm the stiffer member carries its predicted share.

Summary and Next Steps

Key Concepts Mastered

1. **Parallel compound bars** have equal deformation in all materials. Load splits by axial stiffness: $F_i = P \cdot EA_i / \sum EA_j$. Each material carries stress $\sigma_i = E_i \varepsilon$ where ε is the common strain.
2. **Series compound bars** carry the same axial force throughout. Stress in each segment is $\sigma_i = F/A_i$. Total elongation is the sum of segment elongations, $\delta_{total} = \sum FL_i/(A_i E_i)$.
3. **Deformation compatibility** is the constraint that locks the two materials together in a parallel system. Violating it is the most common error in compound bar analysis.
4. **Every segment needs its own safety factor check**, because different areas and different yield strengths mean the critical segment is not always the one with the highest stress.

Results at a Glance

Application	Arrangement	Key results	Governing SF
Linear actuator rod (steel 25 mm + alum 30 mm, 20 kN)	Series	$\sigma_1 = 40.7$ MPa, $\sigma_2 = 28.3$ MPa, $\delta_{total} = 0.162$ mm	9.5 (alum)
Steel core + alum tube column (120 kN, 250 mm)	Parallel	$P_s = 74$ kN (62%), $\sigma_s = 104.7$ MPa, $\sigma_a = 36.6$ MPa, $\delta = 0.131$ mm	3.34 (steel)
Stepped tie rod (steel 20 mm + alum 30 mm, 30 kN)	Series	$\sigma_s = 95.5$ MPa, $\sigma_a = 42.4$ MPa, $\delta_{total} = 0.386$ mm	3.67 (steel)

A Note on Tools

All three worked examples required only the formulas $\sigma = F/A$, $\delta = FL/(AE)$, and the load-splitting rule $F_i = P \cdot EA_i / \sum EA_j$. A spreadsheet or a four-function calculator is sufficient for any compound bar with uniform sections. Finite-element software adds value only when the cross-section varies continuously along the length or when stress concentrations at the material interface need to be resolved.

Next, Thermal Stresses and Strains extends the compound bar framework to temperature loading, where no external force is applied but constrained thermal expansion drives internal stresses that can be large enough to yield the softer material.

Comments

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